

JEE/ NEET

handbook



UNITS AND MEASUREMENTS

- The SI system : It is the international system of units. At present internationally accepted for measurement. In this system there are seven fundamental and two supplementary quantities and their corresponding units are:

Quantity	Unit	Symbol
1. Length	metre	m
2. Mass	kilogram	kg
3. Time	second	s
4. Electric current	ampere	A
5. Temperature	kelvin	K
6. Luminous intensity	candela	cd
7. Amount of substance	mole	mol
Supplementary		
1. Plane angle	radian	rad
2. Solid angle	steradian	sr

- Dimensions : These are the powers to which the fundamental units are raised to get the unit of a physical quantity.
- Uses of dimensions
- To check the correctness of a physical relation.
 - To derive relationship between different physical quantities.
 - To convert one system of unit into another.

$$n_1 u_1 = n_2 u_2$$

$$n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$$

- Significant figures : In any measurement, the reliable digits plus the first uncertain digit are known as significant figures.
- Error : It is the difference between the measured value and true value of a physical quantity or the uncertainty in the measurements.
- Absolute error : The magnitude of the difference between the true value and the measured value is called absolute error.

$$\Delta a_1 = \bar{a} - a_1, \Delta a_2 = \bar{a} - a_2, \Delta a_n = \bar{a} - a_n$$

Mean absolute error

$$\Delta \bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + \dots + |\Delta a_n|}{n} = \frac{1}{n} \sum_{i=1}^n |\Delta a_i|$$

- Relative error : It is the ratio of the mean absolute error to its true value or relative error = $\frac{\Delta \bar{a}}{\bar{a}}$
- Percentage error : It is the relative error in per cent.

$$\text{Percentage error} = \left(\frac{\Delta \bar{a}}{a_{\text{mean}}} \right) \times 100\%$$

MOTION IN A STRAIGHT LINE

- Average speed, $V_{\text{av}} = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$
- Average acceleration, $a_{\text{av}} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$
- The area under the velocity-time curve is equal to the displacement and slope gives acceleration.
- If a body falls freely, the distance covered by it in each subsequent second starting from first second will be in the ratio 1 : 3 : 5 : 7 etc.
- If a body is thrown vertically up with an initial velocity u , it takes u/g second to reach maximum height and u/g second to return, if air resistance is negligible.
- If air resistance acting on a body is considered, the time taken by the body to reach maximum height is less than the time to fall back the same height.
- For a particle having zero initial velocity if $s \propto t^\alpha$, where $\alpha > 2$, then particle's acceleration increases with time.
- For a particle having zero initial velocity if $s \propto t^\alpha$, where $\alpha < 0$, then particle's acceleration decreases with time.
- Kinematic equations :
 $v = u + a_t(t)$; $v^2 = u^2 + 2a_t(s)$
 $S = ut + \frac{1}{2}a_t(t)^2$; $S_n = u + \frac{a}{2}(2n-1)$
 applicable only when $|\vec{a}_t| = a_t$ is constant.
 a_t = magnitude of tangential acceleration, S = distance
 If acceleration is variable use calculus approach.
- Relative velocity : $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

MOTION IN A PLANE

- If T is the time of flight, h maximum height, R horizontal range of a projectile, α its angle of projection, then the relations among these quantities.
- $$h = \frac{gT^2}{8} \quad \dots\dots (1);$$
- $$gT^2 = 2R \tan \alpha \quad \dots\dots (2);$$
- $$R \tan \alpha = 4h \quad \dots\dots (3)$$

- $T = \frac{2u \sin \theta}{g}; h = \frac{u^2 \sin^2 \theta}{2g}$
- $R = \frac{u^2 \sin 2\theta}{g}; R_{\max} = \frac{u^2}{g}$ when $\theta = 45^\circ$
- For a given initial velocity, to get the same horizontal range, there are two angles of projection α and $90^\circ - \alpha$.
- The equation to the parabola traced by a body projected horizontally from the top of a tower of height y , with a velocity u is $y = gx^2/2u^2$, where x is the horizontal distance covered by it from the foot of the tower.
- Equation of trajectory is $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$, which is parabola.
- Equation of trajectory of an oblique projectile in terms of range (R) is $y = x \tan \theta \left(1 - \frac{x}{R}\right)$
- Maximum height is equal to n times the range when the projectile is launched at an angle $\theta = \tan^{-1}(4n)$.
- In a uniform circular motion, velocity and acceleration are constants only in magnitude. Their directions change.
- In a uniform circular motion, the kinetic energy of the body is a constant. $W = 0, \vec{a} \neq 0, \vec{P} \neq \text{constant}, \vec{L} = \text{constant}$
- Centripetal acceleration, $a_r = \omega^2 r = \frac{v^2}{r} = \omega v$ (always applicable) $\vec{a}_r = \vec{\omega} \times \vec{v}$

LAWS OF MOTION

- Newton's first law of motion or law of inertia : It is resistance to change.
- Newton's second law : $\vec{F} = m\vec{a}, \vec{F} = d\vec{p}/dt$
- Impulse : $\Delta \vec{p} = \vec{F}\Delta t, p_2 - p_1 = \int_1^2 \vec{F} dt$
- Newton's third law : $\vec{F}_{12} = -\vec{F}_{21}$
- Frictional force $f_s \leq (f_s)_{\max} = \mu_s R; f_k = \mu_k R$
- Circular motion with variable speed. For complete circles, the string must be taut in the highest position, $u^2 \geq 5ga$.
Circular motion ceases at the instant when the string becomes slack, i.e., when $T = 0$, range of values of u for which the string does go slack is $\sqrt{2ga} < u < \sqrt{5ga}$.
- Conical pendulum : $\omega = \sqrt{g/h}$ where h is height of a point of suspension from the centre of circular motion.
- The acceleration of a lift
 $a = \frac{\text{actual weight} - \text{apparent weight}}{\text{mass}}$
If 'a' is positive lift is moving down, and if it is negative the lift is moving up.

- On a banked road, the maximum permissible speed

$$V_{\max} = \sqrt{\frac{Rg}{\cos \theta} \frac{u_s + \tan \theta}{1 - u_s \tan \theta}}$$

WORK, ENERGY AND POWER

- Work done $W = FS \cos \theta$
- Relation between kinetic energy E and momentum, $P = \sqrt{2mE}$
 $K.E. = \frac{1}{2} mV^2; P.E. = mgh$
- If a body moves with constant power, its velocity (v) is related to distance travelled (x) by the formula $v \propto x^{3/2}$.
- Power $P = \frac{W}{t} = F.V$
- Work due to kinetic force of friction between two contact surfaces is always negative. It depends on relative displacement between contact surfaces.
 $W_{FK} = -F_K(S_{\text{rel}})$.
- $\Sigma W = \Sigma \Delta K, \Sigma W \Rightarrow$ total work due to all kinds of forces, $\Sigma \Delta K \Rightarrow$ total change in kinetic energy.
- $\Sigma W_{\text{conservative}} = -\Sigma \Delta U; \Sigma W_{\text{conservative}} \Rightarrow$ Total work due to all kinds of conservative forces.
 $\Sigma \Delta U \Rightarrow$ Total change in all kinds of potential energy.
- Coefficient of restitution $e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$
- The total momentum of a system of particles is a constant in the absence of external forces.

SYSTEM OF PARTICLES & ROTATIONAL MOTION

- The centre of mass of a system of particles is defined as the point whose position vector is $\vec{R} = \frac{\sum m_i \vec{r}_i}{M}$
- The angular momentum of a system of n particles about the origin is $\vec{L} = \sum_{i=1}^n \vec{r}_i \times \vec{p}_i; L = mvr = I\omega$
- The torque or moment of force on a system of n particles about the origin is $\vec{\tau} = \sum_i \vec{r}_i \times \vec{F}_i$
- The moment of inertia of a rigid body about an axis is defined by the formula $I = \sum m_i r_i^2$
- The kinetic energy of rotation is $K = \frac{1}{2} I\omega^2$
- The theorem of parallel axes : $I_z' = I_z + Ma^2$
Theorem of perpendicular axes : $I_z = I_x + I_y$
- For rolling motion without slipping $v_{\text{cm}} = R\omega$. The kinetic energy of such a rolling body is the sum of kinetic energies of translation and rotation :

$$K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$$

- A rigid body is in mechanical equilibrium if
 - (a) It is translational equilibrium i.e., the total external force on it is zero : $\Sigma F_i = 0$.
 - (b) It is rotational equilibrium i.e., the total external torque on it is zero : $\Sigma \tau_i = \Sigma r_i \times F_i = 0$.
- If a body is released from rest on rough inclined plane,

$$\text{then for pure rolling } \mu_r \geq \frac{n}{n+1} \tan \theta \quad (I_c = nmr^2)$$

$$\text{Rolling with sliding } 0 < \mu_s < \left(\frac{n}{n+1} \right) \tan \theta ;$$

$$\frac{g \sin \theta}{n+1} < a < g \sin \theta$$

GRAVITATION

- Newton's universal law of gravitation
Gravitational force $F = \frac{Gm_1m_2}{r^2}$
 $G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$
- The acceleration due to gravity.
 - (a) at a height h above the Earth's surface
$$g(h) = \frac{GM_E}{(R_E + h)^2} = g \left(1 - \frac{2h}{R_E} \right) \text{ for } h \ll R_E$$

$$g(h) = g(0) \left(1 - \frac{2h}{R_E} \right) \text{ where } g(0) = \frac{GM_E}{R_E^2}$$
 - (b) at depth d below the Earth's surface is
$$g(d) = \frac{GM_E}{R_E^2} \left(1 - \frac{d}{R_E} \right) = g(0) \left(1 - \frac{d}{R_E} \right)$$
 - (c) with latitude λ $g^1 = g - R\omega^2 \cos^2 \lambda$
- Gravitational potential $V_g = -\frac{GM}{r}$
- Intensity of gravitational field $I = \frac{GM}{r^2}$
- The gravitational potential energy
$$V = -\frac{Gm_1m_2}{r} + \text{constant}$$
- The escape speed from the surface of the Earth is
$$v_e = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E} \text{ and has a value of } 11.2 \text{ km s}^{-1}.$$
- Orbital velocity, $v_{orbi} = \sqrt{\frac{GM_E}{R_E}} = \sqrt{gR_E}$
- A geostationary (geosynchronous communication) satellite moves in a circular orbit in the equatorial plane at an approximate distance of $4.22 \times 10^4 \text{ km}$ from the Earth's centre.

- Kepler's 3rd law of planetary motion.

$$T^2 \propto a^3 ; \quad \frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

MECHANICAL PROPERTIES OF SOLIDS

- Hooke's law : stress \propto strain
- Young's modulus of elasticity $Y = \frac{F \Delta \ell}{A \ell}$
- Compressibility = $\frac{1}{\text{Bulk modulus}}$
- $Y = 3k(1 - 2\sigma)$
- $Y = 2n(1 + \sigma)$
- If S is the stress and Y is Young's modulus, the energy density of the wire E is equal to $S^2/2Y$.
- If α is the longitudinal strain and E is the energy density of a stretched wire, Y Young's modulus of wire, then E is equal to $\frac{1}{2}Y\alpha^2$
- Thermal stress = $\frac{F}{A} = Y \alpha \Delta \theta$

MECHANICAL PROPERTIES OF FLUIDS

- Pascal's law : A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel. Pressure exerted by a liquid column $P = h\rho g$
- Bernoulli's principle $P + \rho v^2/2 + \rho gh = \text{constant}$
- Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of interface.
- Stokes' law states that the viscous drag force F on a sphere of radius a moving with velocity v through a fluid of viscosity η $F = -6\pi\eta av$.
- Terminal velocity $V_T = \frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta}$
- The surface tension of a liquid is zero at boiling point. The surface tension is zero at critical temperature.
- If a drop of water of radius R is broken into n identical drops, the work done in the process is $4\pi R^2 S(n^{1/3} - 1)$ and fall in temperature $\Delta \theta = \frac{3T}{J} \sqrt{\frac{1}{r} - \frac{1}{R}}$
- Two capillary tubes each of radius r are joined in parallel. The rate of flow is Q . If they are replaced by single capillary tube of radius R for the same rate of flow, then $R = 2^{1/4} r$.
- Ascent of a liquid column in a capillary tube
$$h = \frac{2s \cos \theta}{r \rho g}$$

➤ Coefficient of viscosity, $n = -\frac{F}{A\left(\frac{dv}{dx}\right)}$

➤ Velocity of efflux $V = \sqrt{2gh}$

THERMAL PROPERTIES OF MATTER

- Relation between different temperature scales :

$$\frac{C}{100} = \frac{F - 32}{100} = \frac{K - 273}{100}$$

- The coefficient of linear expansion (α_ℓ), superficial (β) and volume expansion (α_v) are defined by the relations :

$$\frac{\Delta \ell}{\ell} = \alpha_\ell \Delta T ; \frac{\Delta A}{A} = \beta \Delta T ; \frac{\Delta V}{V} = \alpha_v \Delta T$$

$$\alpha_v = 3\alpha_\ell ; \beta = 2\alpha_\ell$$

- In conduction, heat is transferred between neighbouring parts of a body through molecular collisions, without any

flow of matter. The rate of flow of heat $H = KA \frac{T_C - T_D}{L}$,

where K is the thermal conductivity of the material of the bar.

- Convection involves flow of matter within a fluid due to unequal temperatures of its parts.

- Radiation is the transmission of heat as electromagnetic waves.

- Stefan's law of radiation : $E = \sigma T^4$, where the constant σ is known as Stefan's constant $= 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.

- Wein's displacement law : $\lambda_m T = \text{constant}$, where constant is known as Wein's constant $= 2.898 \times 10^{-3} \text{ m K}$.

- Newton's law of cooling: $\frac{dQ}{dt} = -k(T_2 - T_1)$; where T_1 is the temperature of the surrounding medium and T_2 is the temperature of the body.

- Heat required to change the temperature of the substance, $Q = mc\Delta\theta$

c = specific heat of the substance

- Heat absorbed or released during state change $Q = mL$

L = latent heat of the substance

- Mayer's formula $c_p - c_v = R$

THERMODYNAMICS

- First law of thermodynamics: $\Delta Q = \Delta U + \Delta W$, where ΔQ is the heat supplied to the system, ΔW is the work done by the system and ΔU is the change in internal energy of the system.

- In an isothermal expansion of an ideal gas from volume V_1 to V_2 at temperature T the heat absorbed (Q) equals the work done (W) by the gas, each given by

$$Q = W = nRT \ln \left(\frac{V_2}{V_1} \right)$$

- In an adiabatic process of an ideal gas $PV^\gamma = TV^{\gamma-1}$
 $= \frac{T^\gamma}{P^{\gamma-1}} = \text{constant}$, where $\gamma = \frac{C_p}{C_v}$.

- Work done by an ideal gas in an adiabatic change of state from (P_1, V_1, T_1) to (P_2, V_2, T_2) is $W = \frac{nR(T_1 - T_2)}{\gamma - 1}$

- The efficiency of a Carnot engine is given by $\eta = 1 - \frac{T_2}{T_1}$

- **Second law of thermodynamics:** No engine operating between two temperatures can have efficiency greater than that of the Carnot engine.

- Entropy or disorder $S = \frac{\delta Q}{T}$

KINETIC THEORY

- Ideal gas equation $PV = nRT$

- Kinetic theory of an ideal gas gives the relation $P = \frac{1}{3} nm\bar{v}^2$,

Combined with the ideal gas equation it yields a kinetic interpretation of temperature.

$$\frac{1}{2} nm\bar{v}^2 = \frac{3}{2} k_B T, v_{rms} = (\bar{v}^2)^{1/2} = \sqrt{\frac{3k_B T}{m}}$$

- The law of equipartition of energy is stated thus: the energy for each degree of freedom in thermal equilibrium is $1/2 (k_B T)$

- The translational kinetic energy $E = \frac{3}{2} k_B NT$. This leads

to a relation $PV = \frac{2}{3} E$.

- Degree of freedom : Number of directions in which it can move freely.

- Root mean square (rms) velocity of the gas

$$C = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

- Most probable speed $V_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2KT}{m}}$

- Mean free path $\lambda = \frac{KT}{\sqrt{2} \pi d^2 P}$

OSCILLATIONS

- Displacement in SHM : $Y = a \sin wt$ or, $y = a \cos wt$

- The particle velocity and acceleration during SHM as functions of time are given by, $v(t) = -\omega A \sin(\omega t + \phi)$ (velocity), $a(t) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x(t)$ (acceleration) Velocity amplitude $v_m = \omega A$ and acceleration amplitude $a_m = \omega^2 A$.

- A particle of mass m oscillating under the influence of a Hooke's law restoring force given by $F = -kx$ exhibits simple

harmonic motion with $\omega = \sqrt{\frac{k}{m}}$ (angular frequency),

$$T = 2\pi \sqrt{\frac{m}{k}} \text{ (period)}$$

Such a system is also called a linear oscillator.

- Time period for conical pendulum $T = 2\pi \sqrt{\left(\frac{\ell \cos \theta}{g}\right)}$ where θ angle between string & vertical.

- Energy of the particle $E = k + u = \frac{1}{2} m \omega^2 A^2$

WAVES

- The displacement in a sinusoidal wave $y(x, t) = a \sin(kx - \omega t + \phi)$ where ϕ is the phase constant or phase angle.

- Equation of plane progressive wave : $= a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$

- Equation of stationary wave : $Y = 2a \sin \frac{2\pi t}{T} \cos \frac{2\pi x}{\lambda}$

- The speed of a transverse wave on a stretched string $v = \sqrt{T/\mu}$.

- Sound waves are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed v of sound wave in a fluid having bulk modulus B and density μ is $v = \sqrt{B/\rho}$.

- The speed of longitudinal waves in a metallic bar is $v = \sqrt{Y/\rho}$

For gases, since $B = \gamma P$, the speed of sound is $v = \sqrt{\gamma P/\rho}$

- The interference of two identical waves moving in opposite directions produces standing waves. For a string with fixed ends, standing wave $y(x, t) = [2a \sin kx] \cos \omega t$

- The separation between two consecutive nodes or antinodes is $\lambda/2$.

- A stretched string of length L fixed at both the ends vibrates with frequencies $f = \frac{1}{2} \frac{v}{2L}$.

The oscillation mode with lowest frequency is called the fundamental mode or the first harmonic.

- A pipe of length L with one end closed and other end open (such as air columns) vibrates with frequencies given by

$f = \left(n + \frac{1}{2}\right) \frac{v}{2L}$, $n = 0, 1, 2, 3, \dots$. The lowest frequency given by $v/4L$ is the fundamental mode or the first harmonic.

Open organ pipe $n_1 : n_2 : n_3 : \dots = 1, 2, 3, \dots$, $n = \frac{v}{2L}$

- Beats arise when two waves having slightly different frequencies, f_1 and f_2 and comparable amplitudes, are superposed. The beat frequency $f_{\text{beat}} = f_1 - f_2$

- The Doppler effect is a change in the observed frequency of a wave when the source S and the observer O moves relative to the medium. $f = f_0 \left(\frac{v \pm v_0}{v \pm v_s} \right)$

ELECTROSTATICS

- Coulomb's Law : $\vec{F}_{21} = \text{force on } q_2 \text{ due to } q_1 = \frac{k(q_1 q_2)}{r_{21}^2} \hat{r}_{21}$

where $k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$

- Electric field due to a point charge q has a magnitude $|q|/4\pi\epsilon_0 r^2$

- Field of an electric dipole in its equatorial plane

$$E = \frac{-\vec{p}}{4\pi\epsilon_0 (a^2 + r^2)^{3/2}} \cong \frac{-\vec{p}}{4\pi\epsilon_0 r^3}, \quad \text{for } r \gg a$$

Dipole electric field on the axis at a distance r from the centre:

$$\vec{E} = \frac{2\vec{p}r}{4\pi\epsilon_0 (r^2 - a^2)^2} \cong \frac{2\vec{p}}{4\pi\epsilon_0 r^3} \quad \text{for } r \gg a$$

Dipole moment $\vec{p} = q2a$

- In a uniform electric field \vec{E} , a dipole experiences a torque $\vec{\tau}$ given by $\vec{\tau} = \vec{p} \times \vec{E}$ but experiences no net force.

The flux $\Delta\phi$ of electric field \vec{E} through a small area element

$\Delta\vec{S}$ is given by $\Delta\phi = \vec{E} \cdot \Delta\vec{S}$

- Gauss's law: The flux of electric field through any closed surface S is $1/\epsilon_0$ times the total charge enclosed i.e., Q

- Thin infinitely long straight wire of uniform linear charge density λ : $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{n}$

- Infinite thin plane sheet of uniform surface charge density σ
 $\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$

- Thin spherical shell of uniform surface charge density σ :

$$\vec{E} = \frac{\sigma}{4\pi\epsilon_0 r^2} \hat{r} \quad (r \geq R); \quad \vec{E} = 0 \quad (r < R)$$

- Electric Potential : $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$.

- An equipotential surface is a surface over which potential has a constant value.

- Potential energy of two charges q_1, q_2 at \vec{r}_1, \vec{r}_2 is given by

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}, \quad \text{where } r_{12} \text{ is distance between } q_1 \text{ and } q_2.$$

- Capacitance $C = Q/V$, where Q = charge and V = potential difference

- For a parallel plate capacitor (with vacuum between the plates), $C = \epsilon_0 \frac{A}{d}$.

- The energy U stored in a capacitor of capacitance C , with

$$\text{charge } Q \text{ and voltage } V \text{ is } U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

- For capacitors in the series combination,

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

In the parallel combination, $C_{\text{eq}} = C_1 + C_2 + C_3 + \dots$ where C_1, C_2, C_3, \dots are individual capacitances.

CURRENT ELECTRICITY

- Electric current, $I = \frac{q}{t}$

- Current density j gives the amount of charge flowing per second per unit area normal to the flow, $\vec{J} = nq\vec{v}_d$

- Mobility, $\mu = \frac{V_d}{E}$ and $V_d = \frac{I}{Ane}$
- Resistance $R = \rho \frac{\ell}{A}$, ρ = resistivity of the material
- Equation $\vec{E} = \rho \vec{J}$ another statement of Ohm's law, ρ = resistivity of the material.
- Ohm's law $I \propto V$ or $V = RI$
- (a) Total resistance R of n resistors connected in series $R = R_1 + R_2 + \dots + R_n$
- (b) Total resistance R of n resistors connected in parallel $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$.
- Kirchhoff's Rules – (a) Junction rule: At any junction of circuit elements, the sum of currents entering the junction must equal the sum of currents leaving it.
- (b) Loop rule: The algebraic sum of changes in potential around any closed loop must be zero.
- The Wheatstone bridge is an arrangement of four resistances R_1, R_2, R_3, R_4 . The null-point condition is given by $\frac{R_1}{R_2} = \frac{R_3}{R_4}$
- The potentiometer is a device to compare potential differences. The device can be used to measure potential difference; internal resistance of a cell and compare emf's of two sources. Internal resistance $r = R \left(\frac{\ell_1}{\ell_2} - 1 \right)$
- RC circuit : During charging : $q = CE (1 - e^{-t/RC})$
- During discharging : $q = q_0 e^{-t/RC}$
- According to Joule's Heating law, $H = I^2 R t$

MAGNETISM

- The total force on a charge q moving with velocity v i.e., Lorentz force. $\vec{F} = q (\vec{v} \times \vec{B} + \vec{E})$.
- A straight conductor of length ℓ and carrying a steady current I experiences a force \vec{F} in a uniform external magnetic field \vec{B} , $\vec{F} = I \vec{\ell} \times \vec{B}$, the direction of $\vec{\ell}$ is given by the direction of the current.
- Biot-Savart law $d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \vec{r}}{r^3}$.
- The magnitude of the magnetic field due to a circular coil of radius R carrying a current I at an axial distance x from the centre is $B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$.
- The magnitude of the field B inside a long solenoid carrying a current I is : $B = \mu_0 n I$. For a toroid one obtains, $B = \frac{\mu_0 N I}{2\pi r}$.
- Ampere's Circuital Law : $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I$, where I refers to the current passing through S .

- Force between two long parallel wires $F = \frac{\mu_0 I_1 I_2}{2\pi a} \text{ Nm}^{-1}$.
The force is attractive if currents are in the same direction and repulsive currents are in the opposite direction.
- For current carrying coil $\vec{M} = NI\vec{A}$; torque = $\vec{\tau} = \vec{M} \times \vec{B}$
- Conversion of (i) galvanometer into ammeter, $S = \left(\frac{I_g}{I - I_g} \right) G$
- (ii) galvanometer into voltmeter, $S = \frac{V}{I_g} - G$
- The magnetic intensity, $\vec{H} = \frac{\vec{B}_0}{\mu_0}$.
- The magnetisation \vec{M} of the material is its dipole moment per unit volume. The magnetic field B in the material is, $\vec{B} = \mu_0 (\vec{H} + \vec{M})$
- For a linear material $\vec{M} = \chi \vec{H}$. So that $\vec{B} = \mu \vec{H}$ and χ is called the magnetic susceptibility of the material.
 $\mu = \mu_0 \mu_r$; $\mu_r = 1 + \chi$.

ELECTROMAGNETIC INDUCTION

- The magnetic flux $\phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$, where θ is the angle between \vec{B} & \vec{A} .
- Faraday's laws of induction : $\varepsilon = -N \frac{d\phi_B}{dt}$
- Lenz's law states that the polarity of the induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produces it.
- The induced emf (motional emf) across ends of a rod $\varepsilon = B\ell v$
- The self-induced emf is given by, $\varepsilon = -L \frac{dI}{dt}$
 L is the self-inductance of the coil.
 $L = \frac{\mu_0 N^2 A}{\ell}$
- A changing current in a coil (coil 2) can induce an emf in a nearby coil (coil 1).
 $\varepsilon_1 = -M_{12} \frac{dI_2}{dt}$, M_{12} = mutual inductance of coil 1 w.r.t coil 2
 $M = \frac{\mu_0 N_1 N_2 A}{\ell}$
- Growth of current in an inductor, $i = i_0 [1 - e^{-Rt/L}]$
For decay of current, $i = i_0 e^{-Rt/L}$

ALTERNATING CURRENT

- For an alternating current $i = i_m \sin \omega t$ passing through a resistor R , the average power loss P (averaged over a cycle) due to joule heating is $(1/2) i_m^2 R$.
E.m.f, $E = E_0 \sin \omega t$

- Root mean square (rms) current $I = \frac{i_m}{\sqrt{2}} = 0.707 i_m$.

$$E_{\text{rms}} = \frac{E_0}{\sqrt{2}}$$

- The average power loss over a complete cycle $P = VI \cos \phi$. The term $\cos \phi$ is called the power factor.
- An ac voltage $v = v_m \sin \omega t$ applied to a pure inductor L , drives a current in the inductor $i = i_m \sin (\omega t - \pi/2)$, where $i_m = v_m/X_L$. $X_L = \omega L$ is called inductive reactance.
- An ac voltage $v = v_m \sin \omega t$ applied to a capacitor drives a current in the capacitor: $i = i_m \sin (\omega t + \pi/2)$. Here,
- $$i_m = \frac{v_m}{X_C}, X_C = \frac{1}{\omega C} \text{ is called capacitive reactance.}$$
- An interesting characteristic of a series RLC circuit is the phenomenon of resonance. The circuit exhibits resonance, i.e., the amplitude of the current is maximum at the resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}}$ ($X_L = X_C$).
- Impedance $z = \sqrt{R^2 + (x_L - x_C)^2}$
- Transformation ratio, $K = \frac{N_S}{N_P} = \frac{E_S}{E_P} = \frac{I_P}{I_S}$
- Step up transformer : $N_S > N_P$; $E_S > E_P$; $I_P > I_S$
- Step down transformer $N_P > N_S$; $E_P > E_S$ and $I_P < I_S$
- The quality factor Q defined by $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$ is an indicator of the sharpness of the resonance, the higher value of Q indicating sharper peak in the current.

RAY OPTICS

- Reflection is governed by the equation $\angle i = \angle r'$ and refraction by the Snell's law, $\sin i / \sin r = n$, where the incident ray, reflected ray, refracted ray and normal lie in the same plane.
- Mirror equation: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$
- $$\text{Magnification } M = \frac{V}{u} = \frac{I}{O}$$
- Prism Formula $n_{21} = \frac{n_2}{n_1} = \frac{\sin [(A + D_m)/2]}{\sin (A/2)}$, where D_m is the angle of minimum deviation.
- Dispersion is the splitting of light into its constituent colours. The deviation is maximum for violet and minimum for red.
- Scattering $\propto \frac{1}{\lambda^4}$
- Dispersive power $w = \frac{d_v - d_r}{d}$, where d_v, d_r are deviation of violet and red and d the deviation of mean ray (usually yellow).

- For refraction through a spherical interface (from medium 1 to 2 of refractive index n_1 and n_2 , respectively)

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

- Refractive index of a medium $m = \frac{C}{V}$ ($C = 3 \times 10^8 \text{ m/s}$)

$$r = \frac{1}{\sin C} \text{ (C = Critical angle)}$$

- Condition for TIR : 1. Ray of light must travel from denser to rarer medium 2. Angle of incidence in denser medium > critical angle.

- Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

- Lens maker's formula : $\frac{1}{f} = \frac{(n_2 - n_1)}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

- The power of a lens $P = 1/f$. The SI unit for power of a lens is dioptre (D): $1 \text{ D} = 1 \text{ m}^{-1}$.

- If several thin lenses of focal length f_1, f_2, f_3, \dots are in contact, the effective focal $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots$

- The total power of a combination of several lenses $P = P_1 + P_2 + P_3 + \dots$

- Chromatic aberration if satisfying the equation

$$\frac{w_1}{f_1} + \frac{w_2}{f_2} = 0 \text{ or in terms of powers } w_1 P_1 + w_2 P_2 = 0.$$

- For compound microscope $M = \frac{V_0}{u_0} \left(1 + \frac{D}{f_e} \right)$

when final image at D

$$M = \frac{V_0}{u_0} \cdot \frac{D}{f_e} \text{ when final image at infinity.}$$

WAVE OPTICS

- Wavefront : It is the locus of all the particles vibrating in the same phase.
- The resultant intensity of two waves of intensity $I_0/4$ of phase difference ϕ at any points $I = I_0 \cos^2 \left[\frac{\phi}{2} \right]$, where I_0 is the maximum density.
- Intensity $I \propto (\text{amplitude})^2$
- Condition for dark band : $\delta = (2n - 1) \frac{\lambda}{2}$, for bright band : $\delta = m\lambda$
- Fringe width $\beta = \frac{D\lambda}{d}$
- A thin film of thickness t and refractive index μ appears dark by reflection when viewed at an angle of refraction r if $2\mu t \cos r = n\lambda$ ($n = 1, 2, 3$, etc.)
- A single slit of width a gives a diffraction pattern with a central maximum. The intensity falls to zero at angles of $\pm \frac{1}{a}, \pm \frac{2}{a}, \dots$, etc.

- Amplitude of resultant wave $R = \sqrt{a^2 + b^2 + 2ab \cos \phi}$
- Intensity of wave $I = I_1 + I_2 + 2 \sqrt{I_1 I_2} \cos \phi$
- Brewster law : $\mu = \tan i_p$

MODERN PHYSICS

- Energy of a photon $E = h\nu = \frac{hc}{\lambda}$
- Momentum of a photon $P = \frac{h}{\lambda}$
- Einstein's photoelectric equation

$$\frac{1}{2} m v_{\max}^2 = V_0 e = h\nu - f_0 = h(n - n_0)$$
- Mass defect,
 $\Delta M = (Z m_p + (A - Z) m_n) - M; \Delta E_b = \Delta M c^2.$
 $1 \text{ amu} = 931 \text{ MeV}$
- $E_n = -\frac{Z^2}{n^2} \times 13.6 \text{ eV}$ (For hydrogen like atom)
- According to Bohr's atomic model, angular momentum for the electron revolving in stationary orbit, $mvr = nh/2\pi$
- Radius of the orbit of electron $r = \frac{n^2 h^2}{4\pi^2 m k e^2}$
- Bragg's law : $2d \sin \theta = n\lambda.$
- Radius of the nucleus $R = R_0 A^{1/3}$
- Law of radioactive decay : $N = N_0 e^{-\lambda t}.$

$$\text{Activity} = \frac{dN}{dt} = -\lambda N \text{ (unit is Becquerel)}$$
- Half life period, $T_{1/2} = \frac{0.693}{\lambda}$
- Characteristics X-rays : $\lambda_{K\alpha} < \lambda_{L\alpha}$
 Moseley law : $\nu = a(Z - b)^2$
- Pure semiconductors are called 'intrinsic semiconductors'. The presence of charge carriers (electrons and holes) number of electrons (n_e) is equal to the number of holes (n_h).
- The number of charge carriers can be changed by 'doping' of a suitable impurity in pure semiconductors known as extrinsic semiconductors (n-type and p-type).
- In n-type semiconductors, $n_e \gg n_h$ while in p-type semiconductors $n_h \gg n_e$.
- n-type semiconducting Si or Ge is obtained by doping with pentavalent atoms (donors) like As, Sb, P, etc., while p-type Si or Ge can be obtained by doping with trivalent atom (acceptors) like B, Al, In etc.
- In forward bias (n-side is connected to negative terminal of the battery and p-side is connected to the positive), the barrier is decreased while the barrier increases in reverse bias.

- Diodes can be used for rectifying an ac voltage (restricting the ac voltage to one direction).
- Zener diode is one such special purpose diode. In reverse bias, after a certain voltage, the current suddenly increases (breakdown voltage) in a Zener diode. This property has been used to obtain voltage regulation.
- The important transistor parameters for CE-configuration are:

Input resistance

Output resistance

$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B} \right)_{V_{CE}}$$

$$r_o = \left(\frac{\Delta V_{CE}}{\Delta I_C} \right)_{I_B}$$

$$\text{Current amplification factor, } \beta = \left(\frac{\Delta I_C}{\Delta I_B} \right)_{V_{CE}}; \alpha = \left(\frac{\Delta I_C}{\Delta I_E} \right)_{V_{CE}}$$

The voltage gain of a transistor amplifier in common emitter configuration is:

$$A_v = \left(\frac{v_o}{v_i} \right) = \beta \frac{R_C}{R_B}, \text{ where } R_C \text{ and } R_B \text{ are respectively the}$$

resistances in collector and base sides of the circuit.

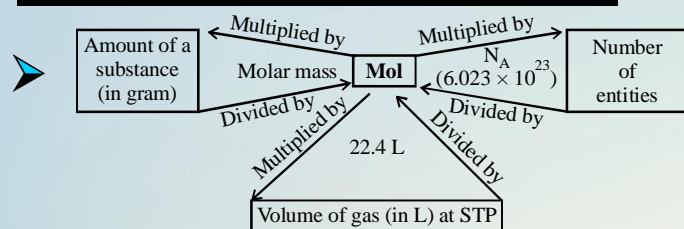
- The important digital circuits performing special logic operations are called logic gates. These are: OR, AND, NOT, NAND, and NOR gates. NAND gate is the combination of NOT and AND gate. NOR gate is the combination of NOT and OR gate.

COMMUNICATION SYSTEMS

- Transmitter, transmission channel and receiver are three basic units of a communication system.
- Two important forms of communication system are: Analog and Digital. The information to be transmitted is generally in continuous waveform for the former while for the latter it has only discrete or quantised levels.
- Low frequencies cannot be transmitted to long distances. Therefore, they are superimposed on a high frequency carrier signal by a process known as modulation.
- In the process of modulation, new frequencies called sidebands are generated on either side.
- If an antenna radiates electromagnetic waves from a height h_T , then the range $d_T = \sqrt{2Rh_T}$ R = radius of earth.
- Effective range, $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$
 h_T = height of transmitting antenna; h_R = height of receiving antenna
- Critical frequency $V_c = 9(N_{\max})^{1/2}$
 where N_{\max} = no. density of electrons/m³
- Skip distance, $D_{\text{skip}} = 2h \left(\frac{V_{\max}}{V_c} \right)^2 - 1$
 h = height of reflecting layer of atmosphere.
- Power radiated by an antenna $\propto \frac{1}{\lambda^2}$

PHYSICAL CHEMISTRY

SOME BASIC CONCEPTS OF CHEMISTRY



- Molecular mass

$$= \frac{\text{Average relative mass of one molecule}}{\frac{1}{12} \times \text{mass of C-12 atom}}$$
- Molecular mass = 2 × VD
- Eq. wt. of metal

$$= \frac{\text{wt. of metal}}{\text{wt. of H}_2 \text{ displaced}} \times 1.008$$
- Eq. wt. of metal = $\frac{\text{wt. of metal}}{\text{wt. of oxygen combined}} \times 8$

$$= \frac{\text{wt. of metal}}{\text{wt. of chlorine combined}} \times 35.5$$
- Molecular formula = (Empirical formula)_n

ATOMIC STRUCTURE

- Energy of electron in species with one electron.

$$E_n = \frac{-2\pi^2 me^4 Z^2}{n^2 h^2}$$

For energy in SI system, $E_n = \frac{-2\pi^2 me^4 Z^2}{n^2 h^2 (4\pi\epsilon_0)^2}$

$$E_n = \frac{-1312Z^2}{n^2} \text{ kJ mol}^{-1}$$

$$mvr = \frac{nh}{2\pi}$$
- $r = \frac{n^2 h^2}{4\pi^2 mZe^2} = 0.529 \left(\frac{n^2}{Z} \right) \text{ \AA}$

- Total energy of electron in the n^{th} shell

$$= \text{K.E.} + \text{P.E.} = kZ \frac{e^2}{2r_n} + \left(-\frac{kZe^2}{r_n} \right) = -\frac{kZe^2}{2r_n}$$
- $\bar{\nu} = \frac{1}{\lambda} = RZ^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right], [R = 1.0968 \times 10^7 \text{ m}^{-1}]$
- $$E = hv = \frac{hc}{\lambda}, \lambda = \frac{h}{\sqrt{2m \times \text{K.E.}}}$$
- No. of spectral lines produced when an electron drops from n^{th} level to ground level = $\frac{n(n-1)}{2}$
- Heisenberg's Uncertainty Principle $(\Delta x)(\Delta p) \geq h/4\pi$
- Nodes $(n-1) = \text{total nodes}$, $\ell = \text{angular nodes}$, $(n-\ell-1) = \text{Radial nodes}$
- Orbital angular momentum : $\sqrt{\ell(\ell+1)} \frac{h}{2\pi} = \sqrt{\ell(\ell+1)} \hbar$

CHEMICAL BONDING

- (i) % ionic character = $\frac{\text{Actual dipole moment}}{\text{Calculated dipole moment}} \times 100$
- (ii) Dipole moment is helpful in predicting geometry and polarity of molecule.
- **Fajan's Rule :** Following factors are helpful in increasing covalent character in ionic compounds
 - (i) Small cation
 - (ii) Big anion
 - (iii) High charge on cation/anion
 - (iv) Cation having pseudo inert gas configuration ($ns^2p^6d^{10}$) e.g. Cu^+ , Ag^+ , Zn^{+2} , Cd^{+2}
- **M.O. theory :**
 - (i) Bond order = $\frac{1}{2}(N_b - N_a)$
 - (ii) Higher the bond order, higher is the bond dissociation energy, greater is the stability, shorter is the bond length.
- Formal charge (F.C.) on an atom in a Lewis structure

$$= [\text{total number of valence electrons in the free atoms}] - [\text{total number of non-binding (lone pair) electrons}] - \frac{1}{2} [\text{total number of bonding (shared) electrons}]$$
- **Relative bond strength :** $sp^3d^2 > dsp^2 > sp^3 > sp^2 > sp > p-p$ (Co-axial) $> s-p > s-s > p-p$ (Co-lateral)
- **VSEPR theory**
 - (i) (LP-LP) repulsion $>$ (LP-BP) $>$ (BP-BP)
 - (ii) $\text{NH}_3 \rightarrow \text{Bond Angle } 106^\circ 45'$ because (LP-BP) repulsion $>$ (BP-BP) $\text{H}_2\text{O} \rightarrow 104^\circ 27'$ because (LP-LP) repulsion $>$ (LP-LB) $>$ (BP-BP)

➤ **Hybridisation :**

$$= \frac{1}{2} \left(\begin{array}{l} \text{number of valence electrons of central atom} \\ + \text{number of monovalent atoms attached to it} \\ + \text{negative charge if any} - \text{positive charge if any} \end{array} \right)$$

CHEMICAL EQUILIBRIUM

➤ $K_p = K_c (RT)^{\Delta n_g}$ where $\Delta n_g = n_p - n_R$

➤ **Free Energy Change (ΔG)**

- (a) If $\Delta G = 0$ then reversible reaction would be in equilibrium, $K_c = 0$
- (b) If $\Delta G = (+)$ ve then equilibrium will be displaced in backward direction; $K_c < 1$
- (c) If $\Delta G = (-)$ ve then equilibrium will shift in forward direction; $K_c > 1$

- (a) K_c unit \rightarrow (moles/lit) $^{\Delta n}$,
- (b) K_p unit \rightarrow (atm) $^{\Delta n}$

➤ **Reaction Quotient and Equilibrium Constant**

Consider the following reversible reaction $A + B \rightleftharpoons C + D$

$$\therefore Q_c = \frac{[C][D]}{[A][B]}$$

Case I : If $Q_c < K_c$ then : [Reactants] > [Products]
then the system is not at equilibrium

Case II : If $Q_c = K_c$ then : The system is at equilibrium.

Case III : If $Q_c > K_c$ then : [Products] > [Reactants]
The system is not at equilibrium.

➤ A relationship between the equilibrium constant K_c , reaction quotient and Gibb's energy.

$$\Delta G = \Delta G^\circ + RT \ln Q$$

At equilibrium $\Delta G = 0$ and $Q = K$ then $\Delta G^\circ = -RT \ln K_c$

$$\therefore \Delta G^\circ = -RT \ln K_p$$

➤ **Le-Chatelier's principle**

- (i) Increase of reactant conc. (Shift reaction forward)
- (ii) Decrease of reactant conc. (Shift reaction backward)
- (iii) Increase of pressure (from more moles to less moles)
- (iv) Decrease of pressure (from less moles to more moles)
- (v) For exothermic reaction decrease in temp. (Shift forward)
- (vi) For endothermic increase in temp. (Shift backward)

- (i) Lewis Acid (e^- pair acceptor) \rightarrow $CO_2, BF_3, AlCl_3, ZnCl_2$, normal cation
- (ii) Lewis Base (e^- pair donor) \rightarrow $NH_3, ROH, ROR, H_2O, RNH_2$, normal anions

➤ **Dissociation of Weak Acid and Weak Base**

- (i) Weak Acid, $K_a = Cx^2/(1-x)$ or $K_a = Cx^2$; $x \ll 1$
- (ii) Weak Base, $K_b = Cx^2/(1-x)$ or $K_b = Cx^2$; $x \ll 1$

➤ **Buffer solution {Henderson equation} :**

- (i) Acidic, $pH = pK_a + \log \{ \text{Salt/Acid} \}$.
For maximum buffer action $pH = pK_a$
Range of buffer $pH = pK_a \pm 1$
- (ii) Alkaline $\rightarrow pOH = pK_b + \log \{ \text{Salt/Base} \}$ for max. buffer action $pH = 14 - pK_b$
Range $pH = 14 - pK_b \pm 1$

$$(iii) \text{ Buffer Capacity} = \frac{\text{Moles / lit of Acid or Base Mixed}}{\text{Change in pH}}$$

➤ **Relation between ionisation constant (K_i) and degree of**

$$\text{ionisation}(\alpha) : - K_i = \frac{\alpha^2}{(1-\alpha)V} = \frac{\alpha^2 C}{(1-\alpha)} = \text{(Ostwald's)}$$

dilution law)

It is applicable to weak electrolytes for which $\alpha \ll 1$ then

$$\alpha = \sqrt{K_i V} = \sqrt{\frac{K_i}{C}} \text{ or } V \uparrow C \downarrow \alpha \uparrow$$

➤ **Common ion effect :** By addition of X mole/L of a common ion, to a weak acid (or weak base) α becomes equal to

$$\frac{K_a}{X} \left(\text{or } \frac{K_b}{X} \right) \text{ [where } \alpha = \text{degree of dissociation}]$$

- (i) If solubility product > ionic product then the solution is unsaturated and more of the substance can be dissolved in it.
- (ii) If ionic product > solubility product the solution is super saturated (principle of precipitation).

➤ **Salt of weak acid and strong base :**

$$pH = 0.5 (pK_w + pK_a + \log c); h = \sqrt{\frac{K_h}{c}}; K_h = \frac{K_w}{K_a}$$

(h = degree of hydrolysis)

Salt of weak base and strong acid :

$$pH = 0.5 (pK_w - pK_b - \log c); h = \sqrt{\frac{K_w}{K_b \times c}}$$

Salt of weak acid and weak base :

$$pH = 0.5 (pK_w + pK_a - pK_b); h = \sqrt{\frac{K_w}{K_a \times K_b}}$$

CHEMICAL KINETICS

➤ **Unit of rate constant :** $k = \text{mol}^{1-n} \text{ lit}^{n-1} \text{ sec}^{-1}$

➤ **Order of reaction** It can be fraction, zero or any whole number.

➤ **Molecularity of reaction** is always a whole number. It is never more than three. It cannot be zero.

➤ **First Order Reactions :**

$$k = \frac{2.303}{t} \log_{10} \frac{a}{(a-x)} \text{ \& } t_{1/2} = \frac{0.693}{k} [A]_t = [A]_0 e^{-kt}$$

➤ **Second Order Reactions :** When concentration of A and B

$$\text{taking same. } k_2 = \frac{1}{t} \left(\frac{x}{a(a-x)} \right)$$

When concentration of A and B are taking different -

$$k_2 = \frac{2.303}{t(a-b)} \log \frac{b(a-x)}{a(b-x)}$$

➤ **Zero Order Reaction :** $x = kt$ and $t_{1/2} = \frac{a}{2k}$

The rate of reaction is independent of the concentration of the reacting substance.

- Time of n^{th} fraction of first order process,

$$t_{1/n} = \frac{2.303}{k} \log \left(\frac{1}{1 - \frac{1}{n}} \right)$$

- Amount of substance left after 'n' half lives = $\frac{[A]_0}{2^n}$

- **Arrhenius equation :** $k = Ae^{-E_a/RT}$, slope = $\frac{-E_a}{2.303R}$

and Temperature Coefficient $\log \left(\frac{k_2}{k_1} \right) = \frac{E_a}{2.303} \left(\frac{T_2 - T_1}{T_1 T_2} \right)$

- It has been found that for a chemical reaction with rise in temperature by 10°C , the rate constant gets nearly doubled.

$$k = PZ_{AB}e^{-E_a/RT}$$

REDOX REACTIONS

- Oxidant itself is reduced (gives O_2)
Or Oxidant $\longrightarrow e^-$ (s) Acceptor
Reductant itself is oxidised (gives H_2)
Or reductant $\longrightarrow e^-$ (s) Donor
- (i) Strength of acid $\propto \text{O.N}$
(ii) Strength of base $\propto 1/\text{O.N}$
- (i) Electrochemical Series:- Li, K, Ba, Sr, Ca, Na, Mg, Al, Mn, Zn, Cr, Fe, Cd, Co, Ni, Sn, Pb, H_2 , Cu, Ag, Pt, Au.
(ii) As we move from top to bottom in this series
(a) Standard Reduction Potential \uparrow
(b) Standard Oxidation Potential
(c) Reducing Capacity \downarrow
(d) IP \uparrow
(e) Reactivity \downarrow

THERMODYNAMICS

- First Law of Thermodynamics : $\Delta E = Q + W$
Expression for pressure volume work $W = -P\Delta V$
Maximum work in a reversible expansion :

$$W = -2.303n RT \log \frac{V_2}{V_1} = -2.303 nRT \log \frac{P_1}{P_2}$$

- $W_{\text{rev}} \geq W_{\text{irr}}$
 $q_v = c_v \Delta T = \Delta U$, $q_p = c_p \Delta T = \Delta H$
Enthalpy changes during phase transformation

- (i) Enthalpy of Fusion
(ii) Heat of Vapourisation
(iii) Heat of Sublimation

- **Enthalpy :** $\Delta H = \Delta E + P\Delta V = \Delta E + \Delta n_g RT$

- **Kirchoff's equation :**

$$\Delta E_{T_2} = \Delta E_{T_1} + \Delta C_V (T_2 - T_1) \text{ [constant V]}$$

$$\Delta H_{T_2} = \Delta H_{T_1} + \Delta C_P (T_2 - T_1) \text{ [constant P]}$$

- **Entropy(s) :** Measure of disorder or randomness
 $\Delta S = \sum S_p - \sum S_R$

$$\Delta S = \frac{q_{\text{rev}}}{T} = 2.303 nR \log \frac{V_2}{V_1} = 2.303 nR \log \frac{P_1}{P_2}$$

- Free energy change : $\Delta G = \Delta H - T\Delta S$, $\Delta G^\circ = -nFE_{\text{cell}}^\circ$
 $-\Delta G = W(\text{maximum}) - P\Delta V$, $\Delta G_{\text{system}} = -T\Delta S_{\text{total}}$

ΔH	ΔS	ΔG	Reaction characteristics
-	+	Always negative	Reaction is spontaneous at all temperature.
+	-	Always positive	Reaction is nonspontaneous at all temperature
-	-	Negative at low temperature but positive at high temperature	Spontaneous at low temp. & non spontaneous at high temperature
+	+	Positive at low temp. but negative at high temperature	Non spontaneous at low temp. & spontaneous at high temp.

ELECTROCHEMISTRY

- $m = ZIt$

- Degree of dissociation : $\alpha = \frac{\lambda_{\text{eq}}}{\lambda_{\text{eq}}^0}$

- Specific conductance

$$\kappa = \frac{1}{\rho} = \frac{\ell}{R \cdot a} = G \times \frac{\ell}{a} = G \times \text{cell constant } (G^*);$$

$$\Lambda_m = \frac{\kappa \times 1000}{M}, \Lambda_{\text{eq}} = \frac{\kappa \times 1000}{N}$$

- Kohlrausch's law : $\Lambda_m^0 = x\lambda_A^0 + y\lambda_B^0$

- Nernst Equation $E = E^\circ - \frac{0.0591}{n} \log_{10} \frac{[\text{Products}]}{[\text{Reactants}]}$

$$\& E^\circ_{\text{Cell}} = E^\circ_{\text{right}} + E^\circ_{\text{left}} \& K_{\text{eq.}} = \text{antilog} \left[\frac{nE^\circ}{0.0591} \right]$$

$$\Delta G = -nFE_{\text{cell}}^\circ \& \Delta G^\circ = -nFE_{\text{cell}}^\circ = -2.303 RT \log K_c$$

$$\& W_{\text{max}} = +nFE^\circ \& \Delta G = \Delta H + T \left(\frac{\partial \Delta G}{\partial T} \right)_P$$

- Calculation of pH of an electrolyte by using a calomel electrode : $\text{pH} = \frac{E_{\text{cell}} - 0.2415}{0.0591}$

- Thermodynamic efficiency of fuel cells : $\eta = \frac{-\Delta G}{\Delta H} = \frac{-nFE_{\text{cell}}^\circ}{\Delta H}$

For $\text{H}_2\text{-O}_2$ fuel cells it is 95%.

- $P = K_H \cdot x$

- Normality (N) = $\frac{\text{number of equivalents}}{\text{volume of the solution in litres}}$

- Molarity (M) = $\frac{\text{number of moles}}{\text{volume of the solution in litres}}$

SOLUTIONS

- Raoult's law

$$P = P_A + P_B = p_A^\circ X_A + p_B^\circ X_B$$

- **Characteristics of an ideal solution:**

- (i) $\Delta_{\text{sol}} V = 0$ (ii) $\Delta_{\text{sol}} H = 0$

- Relative lowering of vapour pressure = $\frac{P_A^o - P_A}{P_A^o}$
- $$\frac{P_A^o - P_A}{P_A^o} = X_B = \frac{n_B}{n_A + n_B}$$
- Colligative \propto Number of particles/ ions/ moles of solute properties
- Depression of freezing point, $\Delta T_f = K_f m$
- Elevation in boiling point with relative lowering of vapour pressure $\Delta T_b = \frac{1000K_b}{M_1} \left(\frac{p^o - p}{p^o} \right)$ (M_1 = mol. wt. of solvent)
- Osmotic pressure (P) with depression in freezing point ΔT_f
- $$P = \Delta T_f \times \frac{dRT}{1000K_f}$$
- Relation between Osmotic pressure and other colligative properties:
- (i) $\pi = \left(\frac{P_A^o - P_A}{P_A^o} \right) \times \frac{dRT}{M_B}$ Relative lowering of vapour pressure
- (ii) $\pi = \Delta T_b \times \frac{dRT}{1000K_b}$ Elevation in boiling point
- (iii) $\pi = \Delta T_f \times \frac{dRT}{1000K_f}$ Depression in freezing point
- $i = \frac{\text{Normal molar mass}}{\text{Observed molar mass}} = \frac{\text{Observed colligative property}}{\text{Normal colligative property}}$
- Degree of association $a = (1 - i) \frac{n}{n - 1}$
- & degree of dissociation (α) = $\frac{i - 1}{n - 1}$

GASEOUS STATE

- Ideal gas equation : $PV = nRT$
- (i) $R = 0.0821 \text{ liter atm. deg}^{-1} \text{ mole}^{-1}$
- (ii) $R = 2 \text{ cal. deg}^{-1} \text{ mole}^{-1}$
- (iii) $R = 8.314 \text{ JK}^{-1} \text{ mole}^{-1}$
- Velocities related to gaseous state
- $$\text{RMS velocity} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{d}}$$
- $$\text{Average speed} = \sqrt{\frac{8RT}{M}} \text{ \& Most probable speed} = \sqrt{\frac{2RT}{M}}$$
- Average speed = $0.9213 \times$ RMS speed
- RMS speed = $1.085 \times$ Average speed
- MPS = $.816 \times$ RMS; RMS = 1.224 MPS
- MPS : A.V. speed : RMS = $1 : 1.128 : 1.224$
- Rate of diffusion $\propto \frac{1}{\sqrt{\text{density of gas}}}$
- van der Waal's equation
- $$\left(P + \frac{n^2 a}{V^2} \right) (V - nb) = nRT \text{ for } n \text{ moles}$$

- Z (compressibility factor) = $\frac{PV}{nRT}$; $Z = 1$ for ideal gas

$$T_C = \frac{8a}{27Rb}, P_C = \frac{a}{27b^2}, V_C = 3b, T_b = \frac{a}{bR}$$

SOLID STATE

- Available space filled up by hard spheres (packing fraction):
- Simple cubic = $\frac{\pi}{6} = 0.52$
- $bcc = \frac{\pi\sqrt{3}}{8} = 0.68$ $fcc = \frac{\pi\sqrt{2}}{6} = 0.74$ $hcp = \frac{\pi\sqrt{2}}{6} = 0.74$
- diamond = $\frac{\pi\sqrt{3}}{6} = 0.34$
- Radius ratio and co-ordination number (CN)
- | Limiting radius ratio | CN | Geometry |
|-----------------------|----|------------------|
| [0.155–0.225] | 3 | [Plane triangle] |
| [0.225–0.414] | 4 | [Tetrahedral] |
| [0.414–0.732] | 6 | [Octahedral] |
| [0.732–1] | 8 | [bcc] |
- Atomic radius r and the edge of the unit cell: Pure elements :
- Simple cubic = $r = \frac{a}{2}$; bcc $r = \frac{\sqrt{3}a}{4}$; $fcc = \frac{\sqrt{2}a}{4}$
- Relationship between radius of void (r) and the radius of the sphere (R) : r (tetrahedral) = $0.225 R$; r (octahedral) = $0.414 R$
- Paramagnetic : Presence of unpaired electrons [attracted by magnetic field]
- Ferromagnetic : Permanent magnetism [$\uparrow\uparrow\uparrow\uparrow$]
- Antiferromagnetic : Net magnetic moment is zero [$\uparrow\downarrow\uparrow\downarrow$]

SURFACE CHEMISTRY

- **Ferrimagnetic** : Net magnetic moment is three [$\uparrow\downarrow\uparrow\uparrow$]
- **Emulsion** : Colloidal soln. of two immiscible liquids [O/W emulsion, W/O emulsion]
- **Emulsifier** : Long chain hydrocarbons are added to stabilize emulsion.
- **Lyophilic colloid** : Starchy gum, gelatin have greater affinity for solvent.
- **Lyophobic colloid** : No affinity for solvent, special methods are used to prepare sol. [e.g. As_2S_3 , $Fe(OH)_3$ sol]
- **Preparation of colloidal solution :**
- (i) Dispersion methods (ii) Condensation method.
- **Coagulating power** $\propto \frac{1}{\text{Flocculating value}}$
- **Properties of colloidal solution :**
- (i) Tyndall effect (ii) Brownian movement
- (iii) Coagulation (iv) Filtrability.

INORGANIC CHEMISTRY

PERIODIC TABLE

- General electronic configuration (of outer orbits)

s-block	ns^{1-2}
p-block	ns^2np^{1-6}
d-block	$(n-1)d^{1-10} ns^{1-2}$
f-block	$(n-2)f^{1-14}s^2p^6d^{10}$ $(n-1)s^2p^6d^0 \text{ or } 1 ns^2$

Property	Pr (L To R)	Gr (T to B)
(i) Atomic radius	↓	↑
(ii) Ionisation potential	↑	↓
(iii) Electron affinity	↑	↓
(iv) Electronegativity	↑	↓
(v) Metallic character or electropositive character	↓	↑
(vi) Alkaline character of hydroxides	↓	
(vii) Acidic character	↑	↓
(viii) Reducing property	↓	↑
(ix) Oxidising property	↑	↓
(x) Non metallic character	↑	↓

➤ $IP \propto \frac{1}{\text{Metallic character}} \propto \frac{1}{\text{Reducing character}}$

➤ $EA \propto \frac{1}{\text{size}} \propto \text{nuclear charge.}$

Second electron affinity is always negative.

Electron affinity of chlorine is greater than fluorine (small atomic size).

- The first element of a group has similar properties with the second element of the next group. This is called diagonal relationship. The diagonal relationship disappears after IV group.

s-BLOCK ELEMENTS

- Atomic radii : $Li < Na < K < Rb < Cs$
- Electronegativity : $Li > Na > K > Rb > Cs$
- First ionization potential : $Li > Na > K > Rb > Cs$
- Melting point : $Li > Na > K > Rb > Cs$
- Colour of the flame : Li - Red, Na - Golden, K - Violet, Rb - Red, Cs - Blue, Ca - Brick red, Sr - Blood red, Ba - Apple green
- Rb and Cs show photoelectric effect.
- Stability of hydrides : $LiH > NaH > KH > RbH > CsH$
- Basic nature of hydroxides : $LiOH < NaOH < KOH < RbOH < CsOH$
- Hydration energy : $Li > Na > K > Rb > Cs$
- Reducing character : $Li > Cs > Rb > K > Na$

BORON FAMILY

- Stability of +3 oxidation state : $B > Al > Ga > In > Tl$
- Stability of +1 oxidation state : $Ga < In < Tl$

Basic nature of the oxides and hydroxides : $B < Al < Ga < In < Tl$
Relative strength of Lewis acid : $BF_3 < BCl_3 < BBr_3 < BI_3$

➤ **Ionisation energy** : $B > Al < Ga > In < Tl$

➤ **Electronegativity** : Electronegativity first decreases from B to Al and then increases marginally.

CARBON FAMILY

- Reactivity : $C < Si < Ge < Sn < Pb$
- Metallic character : $C < Si < Ge < Sn < Pb$
- Acidic character of the oxides : $CO_2 > SiO_2 > GeO_2 > SnO_2 > PbO_2$
Weaker acidic (amphoteric)
- Reducing nature of hydrides : $CH_4 < SiH_4 < GeH_4 < SnH_4 < PbH_4$
- Thermal stability of tetrahalides : $CCl_4 > SiCl_4 > GeCl_4 > SnCl_4 > PbCl_4$
- Oxidising character of M^{+4} species : $GeCl_4 < SnCl_4 < PbCl_4$
- Ease of hydrolysis of tetrahalides : $SiCl_4 < GeCl_4 < SnCl_4 < PbCl_4$

NITROGEN FAMILY

- Acidic strength of trioxides : $N_2O_3 > P_2O_3 > As_2O_3$
- Acidic strength of pentoxides : $N_2O_5 > P_2O_5 > As_2O_5 > Sb_2O_5 > Bi_2O_5$
- Acidic strength of oxides of nitrogen : $N_2O < NO < N_2O_3 < N_2O_4 < N_2O_5$
- Basic nature, bond angle, thermal stability and dipole moment of hydrides : $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$
- Stability of trihalides of nitrogen : $NF_3 > NCl_3 > NBr_3$
- Lewis base strength : $NF_3 < NCl_3 > NBr_3 < NI_3$
- Ease of hydrolysis of trichlorides : $NCl_3 > PCl_3 > AsCl_3 > SbCl_3 > BiCl_3$
- Lewis acid strength of trihalides of P, As and Sb : $PCl_3 > AsCl_3 > SbCl_3$
- Lewis acid strength among phosphorus trihalides : $PF_3 > PCl_3 > PBr_3 > PI_3$
- Nitrogen displays a great tendency to form $p\pi - p\pi$ multiple bonds with itself as well as with carbon and oxygen.
- The basic strength of the hydrides : $NH_3 > PH_3 > AsH_3 > SbH_3$
- The thermal stability of the hydrides decreases as the atomic size increases.

OXYGEN FAMILY

- Melting and boiling point of hydrides : $H_2O > H_2Te > H_2Se > H_2S$
- Volatility of hydrides : $H_2O < H_2Te < H_2Se < H_2S$
- Reducing nature of hydrides : $H_2S < H_2Se < H_2Te$
- Covalent character of hydrides : $H_2O < H_2S < H_2Se < H_2Te$
- The acidic character of oxides (elements in the same oxidation state) : $SO_2 > SeO_2 > TeO_2 > PoO_2$; $SO_3 > SeO_3 > TeO_3$
- Acidic character of oxide of a particular element (e.g. S) : $SO < SO_2 < SO_3$; $SO_2 > TeO_2 > SeO_2 > PoO_2$

HALOGEN FAMILY

- Bond energy of halogens : $\text{Cl}_2 > \text{Br}_2 > \text{F}_2 > \text{I}_2$
- Solubility of halogen in water : $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$
- Oxidising power : $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$
- Enthalpy of hydration of X^- ion : $\text{F}^- > \text{Cl}^- > \text{Br}^- > \text{I}^-$
- Reactivity of halogens : $\text{F} > \text{Cl} > \text{Br} > \text{I}$
- Ionic character of M - X bond in halides
 $\text{M} - \text{F} > \text{M} - \text{Cl} > \text{M} - \text{Br} > \text{M} - \text{I}$
- Reducing character of X^- ion : $\text{I}^- > \text{Br}^- > \text{Cl}^- > \text{F}^-$
- Acidic strength of halogen acids : $\text{HI} > \text{HBr} > \text{HCl} > \text{HF}$
- Conjugate base strength of halogen acids: $\text{I}^- < \text{Br}^- < \text{Cl}^- < \text{F}^-$
- Reducing property of hydrogen halides : $\text{HF} < \text{HCl} < \text{HBr} < \text{HI}$
- Oxidising power of oxides of chlorine
 $\text{Cl}_2\text{O} > \text{ClO}_2 > \text{Cl}_2\text{O}_6 > \text{Cl}_2\text{O}_7$
- acidic character of oxyacids of chlorine
 $\text{HClO} < \text{HClO}_2 < \text{HClO}_3 < \text{HClO}_4$
- Oxidising power of oxyacids of chlorine

NOBLE GASES

- $\text{HClO} > \text{HClO}_2 > \text{HClO}_3 > \text{HClO}_4$
- $\text{XeF}_2 + \text{PF}_5 \longrightarrow [\text{XeF}]^+ [\text{PF}_6]^-$
- $\text{XeF}_4 + \text{SbF}_5 \longrightarrow [\text{XeF}_3]^+ [\text{SbF}_6]^-$
- $\text{XeF}_6 + \text{H}_2\text{O} \longrightarrow \text{XeOF}_4 + 2\text{HF}$
- $\text{XeF}_6 + 2\text{H}_2\text{O} \longrightarrow \text{XeO}_2\text{F}_2 + 4\text{HF}$
- $2\text{XeF}_4 + 3\text{H}_2\text{O} \longrightarrow \text{Xe} + \text{XeO}_3 + 4\text{HF} + \text{F}_2$
- $\text{XeF}_6 + 3\text{H}_2\text{O} \longrightarrow \text{XeO}_3 + 6\text{HF}$
- $2\text{XeF}_6 + \text{SiO}_2 \longrightarrow 2\text{XeOF}_4 + \text{SiF}_4$
- $2\text{XeOF}_4 + \text{SiO}_2 \longrightarrow 2\text{XeO}_2\text{F}_2 + \text{SiF}_4$
- $2\text{XeO}_2\text{F}_4 + \text{SiO}_2 \longrightarrow 2\text{XeO}_3 + \text{SiF}_4$

TRANSITION ELEMENTS (d- and f-BLOCK ELEMENTS)

- The element with exceptional configuration are
 $\text{Cr}^{24}[\text{Ar}] 3d^5 4s^1$, $\text{Cu}^{29}[\text{Ar}] 3d^{10} 4s^1$
 $\text{Mo}^{42}[\text{Kr}] 4d^5 5s^1$, $\text{Pd}^{46}[\text{Kr}] 4d^{10} 5s^0$ $\text{Ag}^{47}[\text{Kr}] 4d^{10} 5s^1$,
 $\text{Pt}^{78}[\text{Xe}] 4f^{14} 5d^{10} 6s^0$
- **Inner Transition Elements**
 - (i) **Electronic Configuration** - $[\text{Xe}] 4f^{0-14} 5d^{0-1} 6s^2$
 - (ii) **Magnetic properties** - Magnetic moment is given by the formula $\mu = \sqrt{4S(S+1) + L(L+1)}$ where L = Orbital quantum number, S = Spin quantum number

COORDINATION COMPOUNDS

- Coordination number is the number of the nearest atoms or groups in the coordination sphere.
- Ligand is a Lewis base donor of electrons that bonds to a central metal atom in a coordination compound.
- Paramagnetic substance is one that is attracted to the magnetic field, this results on account of unpaired electrons present in the atom/molecule/ion.
- Effective atomic number EAN
= (Z - Oxidation number) + (2 × Coordination number)
- Factors affecting stability of complex
 - (i) Greater the charge on the central metal ion, greater is the stability.
 - (ii) Greater the ability of the ligand to donate electron pair (basic strength) greater is the stability.
 - (iii) Formation of chelate rings increases the stability.
- Isomerism in coordination compounds :
 - (i) Structural Isomerism
 - (ii) Ionization Isomerism
 - (iii) Hydration Isomerism
 - (iv) Linkage Isomerism
 - (v) Polymerisation Isomerism
 - (vi) Valence Isomerism
 - (vii) Coordination Position Isomerism
 - (viii) Stereo Isomerism
 - (a) *Geometrical Isomerism*
 - (1) *Square planar complexes of the type*
 MA_2X_2 ; MABX_2 ; MABXY
 - (2) *Octahedral of the type* : MA_4X_2 , $\text{MA}_4\text{X}_2\text{MA}_3\text{X}_3$
 $\text{MA}_2\text{X}_2\text{Y}_2$, $\text{M}(\text{AA})_2\text{X}_2$ and $\text{M}(\text{ABCDEF})$.
 - (b) *Optical isomerism*

ORGANIC CHEMISTRY

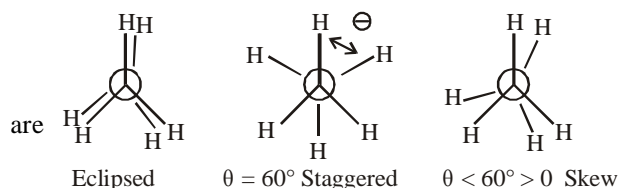
GENERAL ORGANIC CHEMISTRY

- The order of decreasing electronegativity of hybrid orbitals is $sp > sp^2 > sp^3$.
- Conformational isomers are those isomers which arise due to rotation around a single bond.
- A meso compound is optically inactive, even though it has asymmetric centres (due to internal compensation of rotation of plane polarised light)
- An equimolar mixture of enantiomers is called racemic mixture, which is optically inactive.
- Reaction intermediates and reagents :
Homolytic fission → Free radicals

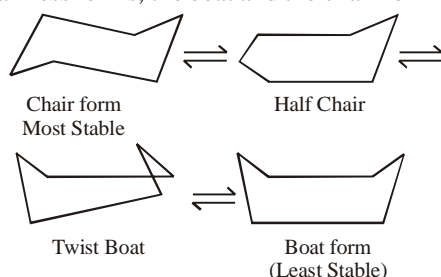
- Heterolytic fission → Ions (Carbonium ions, carbanions etc.)
- Nucleophiles – Electron rich
Two types : (i) Anions (ii) Neutral molecules with lone pair of electrons (Lewis bases)
Electrophiles : Electron deficient.
Two types : (i) Cations (ii) Neutral molecules with vacant orbitals (Lewis acids).
- Inductive effect is due to σ electron displacement along a chain and is permanent effect.
- +I (inductive effect) increases basicity, – I effect increases acidity of compounds.
- Resonance is a phenomenon in which two or more structures can be written for the same compound but none of them actually exists.

ALKANES

- Pyrolytic cracking is a process in which alkane decomposes to a mixture of smaller hydrocarbons, when it is heated strongly, in the absence of oxygen.
- Ethane can exist in an infinite number of conformations. They



- Conformations of Cyclohexane** : It exists in two nonplanar, strainless forms, the boat and the chair form



ALKENES

- In dehydration and dehydrohalogenation the preferential order for removal of hydrogen is $3^\circ > 2^\circ > 1^\circ$ (Saytzeff's rule).
- The lower the ΔH_h (heat of hydrogenation) the more stable the alkene is.
- Alkenes undergo anti-Markonikov addition only with HBr in the presence of peroxides.

ALKYNES

- Alkynes add water molecule in presence of mercuric sulphate and dil. H_2SO_4 and form carbonyl compounds.
- Terminal alkynes have acidic H-atoms, so they form metal alkynides with Na, ammonical cuprous chloride solution and ammoniacal silver nitrate solution.
- Alkynes are acidic because of H-atoms which are attached to sp 'C' atom which has more electronegativity and 's' character than sp^2 and sp^3 'C' atoms.

ARENES

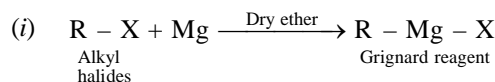
- All o and p-directing groups are ring activating groups (except -X)
They are : -OH, -NH₂, -X, -R, -OR, etc.
- All m-directing groups are ring deactivating groups.
They are : -CHO, -COOH, -NO₂, -CN, -NR₃⁺, etc.

HALOGEN COMPOUNDS

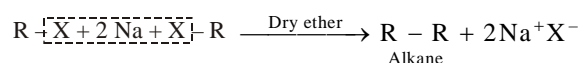
- The order of reactivity is
 - RI > RBr > RCl > RF
 - Allyl halide > Alkyl halide > Vinyl halide
 - Alkyl halide > Aryl halide

- S_N1 reaction** : Mainly 3° alkyl halides undergo this reaction and form racemic mixture. S_N1 is favoured by polar solvent and low concentration of nucleophile.
- S_N2 reaction** : Mainly 1° alkyl halides undergo this substitution. Walden inversion takes place. S_N2 reaction is preferred by non-polar solvents and high concentration of nucleophile.

Reaction with metals:



(ii) Wurtz reaction:



ALCOHOLS

- Alkenes are converted to alcohol in different ways as follows

Reagent	Types of addition
dil H_2SO_4	Markovnikov
B_2H_6 and H_2O_2 , OH ⁻	Anti-Markovnikov
Oxymercuration demercuration	- Markovnikov

- Oxidation of

1° alcohol	→ aldehyde	→ carboxylic acid
	(with same no. of C atom)	(with same no. of C atom)
2° alcohol	→ ketone	→ carboxylic acid
	(with same no. of C atom)	(with less no. of C atom)
3° alcohol	→ ketone	→ carboxylic acid
	(with less no. of C atom)	(with less no. of C atom)

PHENOLS

- Phenol $\xrightarrow{CHCl_3/OH^\ominus}$ Phenolic aldehyde (Reimer-Tieman reaction)
- Phenol $\xrightarrow[\Delta]{CO_2}$ Phenolic carboxylic acid (Kolbe's reaction)
- Acidity of phenols
 - Increases by electron withdrawing substituents like -NO₂, -CN, -CHO, -COOH, -X, -NR₃⁺
 - decreases by electron releasing substituents like -R, -OH, -NH₂, -NR₂, -OR

ETHERS

- $2ROH \xrightarrow[250^\circ C]{Al_2O_3} R-O-R + H_2O$
- $RONa + X-R' \longrightarrow ROR' + NaX$
(Williamson's synthesis)
- $ROR + H_2O \xrightarrow[\Delta]{dil. H_2SO_4} 2ROH$

CARBONYL COMPOUNDS

- Formation of alcohols using RMgX
 - Formaldehyde + RMgX $\xrightarrow{\text{Hydrolysis}}$ 1° alcohol

(b) Aldehyde + $\text{RMgX} \xrightarrow{\text{Hydrolysis}}$ 2° alcohol
(other than HCHO)

(c) Ketone + $\text{RMgX} \xrightarrow{\text{Hydrolysis}}$ 3° alcohol

➤ Cannizzaro reaction (Disproportionation)

Aldehyde $\xrightarrow[\text{alkali}]{\text{Hot conc.}}$ Alcohol + Salt of acid

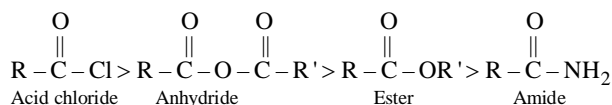
(no α H-atom)

➤ Aldol condensation :

Carbonyl compound + dil. alkali \longrightarrow β -hydroxy carbonyl compound
(with α H-atom)

➤ Benzoin condensation Benzaldehyde $\xrightarrow[\text{NaCN}]{\text{ethanolic}}$ Benzoin

➤ The relative reactivities of different acid derivatives towards nucleophilic acyl substitution reaction follow the order:



CARBOXYLIC ACIDS

➤ The rate of esterification decreases when alcohol, acid or both have branched substituents.

➤ Ortho effect : All ortho substituted benzoic acids (irrespective of type of substituent) are stronger than benzoic acid.

NITROGEN COMPOUNDS

➤ Order of basicity :

($\text{R} = -\text{CH}_3$ or $-\text{C}_2\text{H}_5$) $2^\circ > 1^\circ > 3^\circ > \text{NH}_3$

➤ Hofmann degradation

Amides $\xrightarrow{\text{Br}_2/\text{KOH}}$ 1° amine

➤ The basicity of amines is (i) decreased by electron withdrawing groups (ii) increased by electron releasing groups

➤ Reduction of nitrobenzene in different media gives different products

Medium	Product
Acidic	Aniline
Basic	Azoxy, Azo and finally hydrazobenzene
Neutral	Phenyl hydroxylamine

BIOMOLECULES

➤ Carbohydrates are polyhydroxy aldehydes or ketones.

➤ Monosaccharides are simple sugars, containing three to nine carbon atoms.

➤ Characteristic reactions :

Homologous series

(i) Alkanes

(ii) Alkenes and alkynes

(iii) Arenes

(iv) Alkyl halides

(v) Aldehyde and ketones

Type of reactions

Substitution

Mostly free radical

Electrophilic addition

Electrophilic substitution

Nucleophilic substitution

Nucleophilic addition

➤ Tests to differentiate :

1°, 2° and 3° alcohols

(i) Lucas test

(ii) Victor Meyer's test

1°, 2° and 3° amines

Hinsberg test

1°, 2° and 3° nitro compounds Test with HNO_2 and KOH

Aryl halides and alkyl halides Test with AgNO_3 solution

Aldehydes and ketones Tollen's test/Fehling's test

Aromatic aldehydes and Fehling's test

Aliphatic aldehydes

IMPORTANT REAGENTS

➤ Dil. H_2SO_4 [or Conc. $\text{H}_2\text{SO}_4 + \text{H}_2\text{O}$]

Use \rightarrow Hydrating agent (+ HOH)

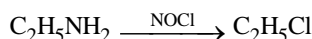
➤ Alc. KOH or NaNH_2 (Use $\rightarrow -\text{HX}$)



➤ Lucas Reagent $\text{ZnCl}_2 + \text{Conc. HCl}$

Use \rightarrow For distinction between 1°, 2° & 3° alc.

➤ Tilden Reagent NOCl (Nitrosyl chloride)

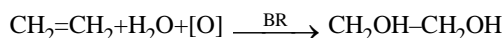


➤ Alkaline KMnO_4 (Strong oxidant)

Toluene \rightarrow Benzoic acid

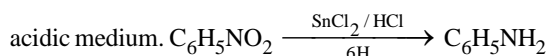
➤ Bayer's Reagent : 1% alkaline KMnO_4 (Weak oxidant)

Use: \rightarrow For test of $>\text{C}=\text{C}<$ or $-\text{C}=\text{C}-$

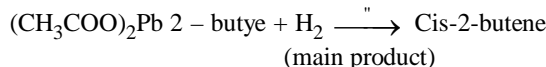


➤ Acidic $\text{K}_2\text{Cr}_2\text{O}_7$ (Strong oxidant) : $\text{RCH}_2\text{OH} \xrightarrow{[\text{O}]}$ RCHO

➤ SnCl_2/HCl or Sn/HCl used for reduction of nitrobenzene in



➤ Lindlar's Catalyst = Pd/CaCO_3 + in small quantity



➤ Ziegler-Natta Catalyst $(\text{C}_2\text{H}_5)_3\text{Al} + \text{TiCl}_4$

Use \rightarrow In Addition polymerisation

IDENTIFICATION TESTS

➤ Unsaturated compound (Bayer's reagent)

Decolourising the reagent

➤ Alcohols (Ceric ammonium nitrate solution)

Red colouration

➤ Phenols (Neutral FeCl_3 solution)

Violet/deep blue colouration

➤ Aldehydes and ketones (2, 4-D.N.P.)

Orange precipitate

➤ Acids (NaHCO_3 solution)

Brisk effervescence (CO_2 is evolved)

➤ 1° amine ($\text{CHCl}_3 + \text{KOH}$)

Foul smell (isocyanide)

➤ 2° amine ($\text{NaNO}_2 + \text{HCl}$)

Yellow oily liquid (Nitrosoamine)

RELATIONS AND FUNCTIONS

- A relation R from a set A to a set B is a subset of the cartesian product $A \times B$ obtained by describing a relationship between the first element x and the second element y of the ordered pairs in $A \times B$.
- **Function :** A function f from a set A to a set B is a specific type of relation for which every element x of set A has one and only one image y in set B . We write $f: A \rightarrow B$, where $f(x) = y$.
- A function $f: X \rightarrow Y$ is one-one (or injective) if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \forall x_1, x_2 \in X$.
- A function $f: X \rightarrow Y$ is onto (or surjective) if given any $y \in Y, \exists x \in X$ such that $f(x) = y$.
- **Many-One Function :**
A function $f: A \rightarrow B$ is called many- one, if two or more different elements of A have the same f - image in B .
- **Into function :**
A function $f: A \rightarrow B$ is into if there exist at least one element in B which is not the f - image of any element in A .
- **Many One -Onto function :**
A function $f: A \rightarrow R$ is said to be many one- onto iff f is onto but not one-one.
- **Many One -Into function :**
A function is said to be many one-into if it is neither one-one nor onto.
- A function $f: X \rightarrow Y$ is invertible if and only if f is one-one and onto.

TRIGONOMETRIC FUNCTIONS AND EQUATIONS

- **General Solution of the equation $\sin \theta = 0$:**
when $\sin \theta = 0$
 $\theta = n\pi; n \in I$ i.e. $n = 0, \pm 1, \pm 2, \dots$
- General solution of the equation $\cos \theta = 0$:**
when $\cos \theta = 0$
 $\theta = (2n+1)\pi/2, n \in I$ i.e. $n = 0, \pm 1, \pm 2, \dots$
- General solution of the equation $\tan \theta = 0$:**
General solution of $\tan \theta = 0$ is $\theta = n\pi; n \in I$
- **General solution of the equation**
 - (a) $\sin \theta = \sin \alpha : \theta = n\pi + (-1)^n \alpha; n \in I$
 - (b) $\sin \theta = k$, where $-1 \leq k \leq 1$.
 $\theta = n\pi + (-1)^n \alpha$, where $n \in I$ and $\alpha = \sin^{-1} k$
 - (c) $\cos \theta = \cos \alpha : \theta = 2n\pi \pm \alpha, n \in I$
 - (d) $\cos \theta = k$, where $-1 \leq k \leq 1$.
 $\theta = 2n\pi \pm \alpha$, where $n \in I$ and $\alpha = \cos^{-1} k$

- (e) $\tan \theta = \tan \alpha : \theta = n\pi + \alpha; n \in I$
- (f) $\tan \theta = k, \theta = n\pi + \alpha$, where $n \in I$ and $\alpha = \tan^{-1} k$
- (g) $\sin^2 \theta = \sin^2 \alpha : \theta = n\pi \pm \alpha; n \in I$
- (h) $\cos^2 \theta = \cos^2 \alpha : \theta = n\pi \pm \alpha; n \in I$
- (i) $\tan^2 \theta = \tan^2 \alpha : \theta = n\pi \pm \alpha; n \in I$

$\sin \alpha + \sin (\alpha + \beta) + \sin (\alpha + 2\beta) + \dots$ to n terms

$$= \frac{\sin \left[\alpha + \left(\frac{n-1}{2} \right) \beta \right] \left[\sin \left(\frac{n\beta}{2} \right) \right]}{\sin (\beta/2)} ; \beta \neq 2n\pi$$

$\cos \alpha + \cos (\alpha + \beta) + \cos (\alpha + 2\beta) + \dots$ to n terms

$$= \frac{\cos \left[\alpha + \left(\frac{n-1}{2} \right) \beta \right] \left[\sin \left(\frac{n\beta}{2} \right) \right]}{\sin (\beta/2)} ; \beta \neq 2n\pi$$

$$\tan \left(\frac{B-C}{2} \right) = \left(\frac{b-c}{b+c} \right) \cot \left(\frac{A}{2} \right)$$

$$\sin \left(\frac{A}{2} \right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\tan \left(\frac{A}{2} \right) = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$R = \frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{c}{2 \sin C}$$

$$R = \frac{abc}{4D}$$

$$r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$a = c \cos B + b \cos C$$

Maximum value of $a \sin \theta + b \cos \theta = \sqrt{a^2 + b^2}$ and minimum value of $a \sin \theta + b \cos \theta = -\sqrt{a^2 + b^2}$

INVERSE TRIGONOMETRIC FUNCTIONS

- **Properties of inverse trigonometric function**

$$\bullet \tan^{-1} x + \tan^{-1} y = \begin{cases} \tan^{-1} \left(\frac{x+y}{1-xy} \right), & \text{if } xy < 1 \\ \pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right), & \text{if } x > 0, y > 0 \text{ and } xy > 1 \\ -\pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right), & \text{if } x < 0, y < 0 \text{ and } xy > 1 \end{cases}$$

$$\bullet \tan^{-1} x - \tan^{-1} y = \begin{cases} \tan^{-1} \left(\frac{x-y}{1+xy} \right), & \text{if } xy > -1 \\ \pi + \tan^{-1} \left(\frac{x-y}{1+xy} \right), & \text{if } x > 0, y < 0 \text{ and } xy < -1 \\ -\pi + \tan^{-1} \left(\frac{x-y}{1+xy} \right), & \text{if } x < 0, y > 0 \text{ and } xy < -1 \end{cases}$$

$$\bullet \sin^{-1} x + \sin^{-1} y =$$

$$\begin{cases} \sin^{-1} \{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}, & \text{if } -1 \leq x, y \leq 1 \text{ and } x^2 + y^2 \leq 1 \\ & \text{or if } xy < 0 \text{ and } x^2 + y^2 > 1 \\ \pi - \sin^{-1} \{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}, & \text{if } 0 < x, y \leq 1 \\ & \text{and } x^2 + y^2 > 1 \\ -\pi - \sin^{-1} \{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}, & \text{if } -1 \leq x, y < 0 \text{ and } x^2 + y^2 > 1 \end{cases}$$

$$\bullet \cos^{-1} x + \cos^{-1} y =$$

$$\begin{cases} \cos^{-1} \{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}, & \text{if } -1 \leq x, y \leq 1 \text{ and } x+y \geq 0 \\ 2\pi - \cos^{-1} \{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}, & \text{if } -1 \leq x, y \leq 1 \text{ and } x+y \leq 0 \end{cases}$$

$$2 \sin^{-1} x = \begin{cases} \sin^{-1} (2x\sqrt{1-x^2}), & \text{if } -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1} (2x\sqrt{1-x^2}), & \text{if } \frac{1}{\sqrt{2}} \leq x \leq 1 \\ -\pi - \sin^{-1} (2x\sqrt{1-x^2}), & \text{if } -1 \leq x \leq -\frac{1}{\sqrt{2}} \end{cases}$$

$$2 \tan^{-1} x = \begin{cases} \tan^{-1} \left(\frac{2x}{1-x^2} \right), & \text{if } -1 < x < 1 \\ \pi + \tan^{-1} \left(\frac{2x}{1-x^2} \right), & \text{if } x > 1 \\ -\pi + \tan^{-1} \left(\frac{2x}{1-x^2} \right), & \text{if } x < -1 \end{cases}$$

QUADRATIC EQUATIONS AND INEQUALITIES

➤ **Roots of a Quadratic Equation :** The roots of the quadratic

$$\text{equation are given by } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Nature of roots : In Quadratic equation $ax^2 + bx + c = 0$. The term $b^2 - 4ac$ is called discriminant of the equation. It is denoted by Δ or D .

(A) Suppose $a, b, c \in \mathbb{R}$ and $a \neq 0$

- (i) If $D > 0 \Rightarrow$ Roots are Real and unequal
- (ii) If $D = 0 \Rightarrow$ Roots are Real and equal and each equal to $-b/2a$
- (iii) If $D < 0 \Rightarrow$ Roots are imaginary and unequal or complex conjugate.

(B) Suppose $a, b, c \in \mathbb{Q}$ and $a \neq 0$

- (i) If $D > 0$ and D is perfect square \Rightarrow Roots are unequal and Rational

(ii) If $D > 0$ and D is not perfect square \Rightarrow Roots are irrational and unequal.

➤ **Condition for Common Root(s)**

Let $ax^2 + bx + c = 0$ and $dx^2 + ex + f = 0$ have a common root α (say).

Condition for both the roots to be common is $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

➤ If $p + iq$ (p and q being real) is a root of the quadratic equation, where $i = \sqrt{-1}$, then $p - iq$ is also a root of the quadratic equation.

➤ Every equation of n^{th} degree ($n \geq 1$) has exactly n roots and if the equation has more than n roots, it is an identity.

COMPLEX NUMBERS

➤ **Exponential Form:** If $z = x + iy$ is a complex number then its exponential form is $z = re^{i\theta}$ where r is modulus and θ is amplitude of complex number.

➤ (i) $|z_1| + |z_2| \geq |z_1 + z_2|$; here equality holds when $\arg(z_1/z_2) = 0$ i.e. z_1 and z_2 are parallel.

(ii) $\|z_1| - |z_2| \leq |z_1 - z_2|$; here equality holds when $\arg(z_1/z_2) = \pi$ i.e. z_1 and z_2 are parallel.

(iii) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

➤ $\arg(z_1 z_2) = \theta_1 + \theta_2 = \arg(z_1) + \arg(z_2)$

➤ $\arg\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2 = \arg(z_1) - \arg(z_2)$

➤ For any integer k , $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$

➤ $|z - z_1| + |z - z_2| = \lambda$, represents an ellipse if $|z_1 - z_2| < \lambda$, having the points z_1 and z_2 as its foci. And if $|z_1 - z_2| = \lambda$, then z lies on a line segment connecting z_1 and z_2 .

➤ **Properties of Cube Roots of Unity**

(i) $1 + \omega + \omega^2 = 0$ (ii) $\omega^3 = 1$

(iii) $1 + \omega^n + \omega^{2n} = 3$ (if n is multiple of 3)

(iv) $1 + \omega^n + \omega^{2n} = 0$ (if n is not a multiple of 3).

PERMUTATIONS AND COMBINATIONS

➤ The number of permutations of n different things, taken r at a time, where repetition is allowed, is n^r .

➤ **Selection of Objects with Repetition :**

The total number of selections of r things from n different things when each thing may be repeated any number of times is ${}^{n+r-1}C_r$

➤ **Selection from distinct objects :**

The number of ways (or combinations) of n different things selecting at least one of them is ${}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n - 1$. This can also be stated as the total number of combination of n different things.

➤ **Selection from identical objects :**

The number of ways to select some or all out of $(p + q + r)$ things where p are alike of first kind, q are alike of second kind and r are alike of third kind is

$$(p + 1)(q + 1)(r + 1) - 1$$

➤ **Selection when both identical and distinct objects are present:**

If out of $(p + q + r + t)$ things, p are alike one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of combinations is

$$(p + 1)(q + 1)(r + 1)2^t - 1$$

➤ **Circular permutations:**

(a) **Arrangements round a circular table :**

The number of circular permutations of n different things

taken all at a time is $\frac{n!}{n} = (n - 1)!$, if clockwise and anticlockwise orders are taken as different.

(b) **Arrangements of beads or flowers (all different) around a circular necklace or garland:**

The number of circular permutations of ' n ' different

things taken all at a time is $\frac{1}{2}(n - 1)!$, if clockwise and anticlockwise orders are taken to be same.

➤ **Sum of numbers :**

(a) For given n different digits $a_1, a_2, a_3, \dots, a_n$ the sum of the digits in the unit place of all numbers formed (if numbers are not repeated) is $(a_1 + a_2 + a_3 + \dots + a_n)(n - 1)!$

(b) Sum of the total numbers which can be formed with given n different digits a_1, a_2, \dots, a_n is $(a_1 + a_2 + a_3 + \dots + a_n)(n - 1)!$. (111 n times)

BINOMIAL THEOREM

➤ **Greatest binomial coefficients :** In a binomial expansion binomial coefficients of the middle terms are called as greatest binomial coefficients.

(a) If n is even : When $r = \frac{n}{2}$ i.e. ${}^nC_{n/2}$ takes maximum value.

(b) If n is odd : $r = \frac{n-1}{2}$ or $\frac{n+1}{2}$

i.e. ${}^nC_{\frac{n-1}{2}} = {}^nC_{\frac{n+1}{2}}$ and take maximum value.

➤ **Important Expansions :**

If $|x| < 1$ and $n \in \mathbb{Q}$ but $n \notin \mathbb{N}$, then

$$(a) \quad (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$

$$(b) \quad (1-x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}(-x)^r + \dots$$

SEQUENCE AND SERIES

➤ **Properties related to A.P. :**

(i) Common difference of AP is given by $d = S_2 - 2S_1$ where S_2 is sum of first two terms and S_1 is sum of first term.

(ii) If for an AP sum of p terms is q , sum of q terms is p , then sum of $(p + q)$ term is $(p + q)$.

(iii) In an A.P. the sum of terms equidistant from the beginning and end is constant and equal to sum of first and last terms.

(iv) If terms $a_1, a_2, \dots, a_n, a_{n+1}, \dots, a_{2n+1}$ are in A.P., then sum of these terms will be equal to $(2n + 1)a_{n+1}$.

(v) If for an A.P. sum of p terms is equal to sum of q terms then sum of $(p + q)$ terms is zero

(vi) Sum of n AM's inserted between a and b is equal to n times the single AM between a and b i.e. $\sum_{r=1}^n A_r = nA$ where $A = \frac{a+b}{2}$

➤ The geometric mean (G.M.) of any two positive numbers a and b is given by \sqrt{ab} i.e., the sequence a, G, b is G.P.

n GM's between two given numbers: If in between two numbers ' a ' and ' b ', we have to insert n GM G_1, G_2, \dots, G_n then $a, G_1, G_2, \dots, G_n, b$ will be in G.P.

The series consist of $(n + 2)$ terms and the last term is b and first term is a .

$$\Rightarrow ar^{n+2-1} = b \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_1 = ar, G_2 = ar^2, \dots, G_n = ar^n \text{ or } G_n = b/r$$

➤ **Use of inequalities in progression :**

(a) Arithmetic Mean \geq Geometric Mean

(b) Geometric Mean \geq Harmonic Mean :

$$A \geq G \geq H$$

STRAIGHT LINES

➤ An acute angle (say θ) between lines L_1 and L_2 with slopes

$$m_1 \text{ and } m_2 \text{ is given by } \tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|, 1 + m_1 m_2 \neq 0$$

➤ Three points A, B and C are collinear, if and only if slope of $AB =$ slope of BC .

➤ The equation of the line having normal distance from origin is p and angle between normal and the positive x -axis is ω , is given by $x \cos \omega + y \sin \omega = p$.

➤ **Co-ordinate of some particular points :**

Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of any triangle ABC , then

Incentre : Co-ordinates of incentre

$$\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$$

where a, b, c are the sides of triangle ABC

- **Area of a triangle :** Let (x_1, y_1) , (x_2, y_2) and (x_3, y_3) respectively be the coordinates of the vertices A, B, C of a triangle ABC. Then the area of triangle ABC, is

$$\frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

Or

$$= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

CONIC SECTIONS

- **Condition of Tangency :** Circle $x^2 + y^2 = a^2$ will touch the line.
- $$y = mx + c \text{ if } c = \pm a\sqrt{1+m^2}$$
- **Pair of Tangents :** From a given point $P(x_1, y_1)$ two tangents PQ and PR can be drawn to the circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$. Their combined equation is $SS_1 = T^2$.
- **Condition of Orthogonality :** If the angle of intersection of the two circle is a right angle ($\theta = 90^\circ$) then such circle are called Orthogonal circle and conditions for their orthogonality is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$
- **Tangent to the parabola :**
- Condition of Tangency :** If the line $y = mx + c$ touches a parabola $y^2 = 4ax$ then $c = a/m$
- **Tangent to the Ellipse :**
- Condition of tangency and point of contact :**
- The condition for the line $y = mx + c$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is that $c^2 = a^2m^2 + b^2$ and the coordinates of the points of contact are $\left(\pm \frac{a^2m}{\sqrt{a^2m^2 + b^2}}, \mp \frac{b^2}{\sqrt{a^2m^2 + b^2}} \right)$
- **Normal to the ellipse**
- (i) **Point Form :** The equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$
- (ii) **Parametric Form :** The equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $(a \cos \theta, b \sin \theta)$ is $ax \sec \theta - by \csc \theta = a^2 - b^2$
- **Tangent to the hyperbola :**
- Condition for tangency and points of contact :** The condition for the line $y = mx + c$ to be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is that $c^2 = a^2m^2 - b^2$ and the coordinates of the points of contact are $\left(\pm \frac{a^2m}{\sqrt{a^2m^2 - b^2}}, \pm \frac{b^2}{\sqrt{a^2m^2 - b^2}} \right)$

- **Chord of contact :**

The equation of chord of contact of tangent drawn from a

point $P(x_1, y_1)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $T = 0$

$$\text{where } T \equiv \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$$

- **Equation of normal in different forms :**

Point Form : The equation of the normal to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ at the point } (x_1, y_1) \text{ is } \frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$$

THREE DIMENSIONAL GEOMETRY

- **Slope Form :** The equation of normal to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ in terms of slope 'm' is } y = mx \pm \frac{m(a^2 + b^2)}{\sqrt{a^2 - b^2m^2}}$$

- **Conditions of Parallelism and Perpendicularity of Two Lines:**

Case-I : When dc's of two lines AB and CD, say ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are known.

$$AB \parallel CD \Leftrightarrow \ell_1 = \ell_2, m_1 = m_2, n_1 = n_2$$

$$AB \perp CD \Leftrightarrow \ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$$

Case-II : When dr's of two lines AB and CD, say a_1, b_1, c_1 and a_2, b_2, c_2 are known

$$AB \parallel CD \Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$AB \perp CD \Leftrightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$$

- If ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are the direction cosines of two lines; and θ is the acute angle between the two lines; then $\cos \theta = |\ell_1\ell_2 + m_1m_2 + n_1n_2|$.

- Equation of a line through a point (x_1, y_1, z_1) and having

$$\text{direction cosines } \ell, m, n \text{ is } \frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

- Shortest distance between $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$

$$\text{is } \frac{|(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|}$$

- Let the two lines be

$$\frac{x - \alpha_1}{\ell_1} = \frac{y - \beta_1}{m_1} = \frac{z - \gamma_1}{n_1} \dots\dots\dots(1)$$

$$\text{and } \frac{x - \alpha_2}{\ell_2} = \frac{y - \beta_2}{m_2} = \frac{z - \gamma_2}{n_2} \dots\dots\dots(2)$$

These lines will coplanar if

$$\begin{vmatrix} \alpha_2 - \alpha_1 & \beta_2 - \beta_1 & \gamma_2 - \gamma_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$$

The plane containing the two lines is

$$\begin{vmatrix} x - \alpha_1 & y - \beta_1 & z - \gamma_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$$

- The equation of a plane through a point whose position vector is \vec{a} and perpendicular to the vector \vec{N} is

$$(\vec{r} - \vec{a}) \cdot \vec{N} = 0$$

- Vector equation of a plane that passes through the intersection of planes $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is

$$\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + \lambda d_2, \text{ where } \lambda \text{ is any nonzero constant.}$$

- Two planes $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are coplanar if $(\vec{a}_2 - \vec{a}_1) + (\vec{b}_1 \times \vec{b}_2) = 0$

DIFFERENTIAL CALCULUS

- **Existence of Limit :**

$$\lim_{x \rightarrow a} f(x) \text{ exists} \Rightarrow \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = \ell$$

Where ℓ is called the limit of the function

- (i) If $f(x) \leq g(x)$ for every x in the deleted nbd of a , then $\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x)$
- (ii) If $f(x) \leq g(x) \leq h(x)$ for every x in the deleted nbd of a and $\lim_{x \rightarrow a} f(x) = \ell = \lim_{x \rightarrow a} h(x)$ then $\lim_{x \rightarrow a} g(x) = \ell$
- (iii) $\lim_{x \rightarrow a} f(g(x)) = f\left(\lim_{x \rightarrow a} g(x)\right) = f(m)$ where $\lim_{x \rightarrow a} g(x) = m$

- (iv) If $\lim_{x \rightarrow a} f(x) = +\infty$ or $-\infty$, then $\lim_{x \rightarrow a} \frac{1}{f(x)} = 0$

CONTINUITY AND DIFFERENTIABILITY OF FUNCTIONS

- A function $f(x)$ is said to be continuous at a point $x = a$ if

$$\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = f(a)$$

- **Discontinuous Functions :**

- (a) **Removable Discontinuity:** A function f is said to have removable discontinuity at $x = a$ if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$ but their common value is not equal to $f(a)$.

- (b) **Discontinuity of the first kind:** A function f is said to have a discontinuity of the first kind at $x = a$ if

$$\lim_{x \rightarrow a^-} f(x) \text{ and } \lim_{x \rightarrow a^+} f(x) \text{ both exist but are not equal.}$$

- (c) **Discontinuity of second kind:** A function f is said to have a discontinuity of the second kind at $x = a$ if neither

$$\lim_{x \rightarrow a^-} f(x) \text{ nor } \lim_{x \rightarrow a^+} f(x) \text{ exists.}$$

Similarly, if $\lim_{x \rightarrow a^+} f(x)$ does not exist, then f is said to have discontinuity of the second kind from the right at $x = a$.

- **For a function f :**

Differentiability \Rightarrow Continuity;

Continuity \nRightarrow derivability

Not derivability \nRightarrow discontinuous ;

But discontinuity \Rightarrow Non derivability

- **Differentiation of infinite series:**

(i) If $y = \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x) + \dots \infty}}}$

$$\Rightarrow y = \sqrt{f(x) + y} \Rightarrow y^2 = f(x) + y$$

$$2y \frac{dy}{dx} = f'(x) + \frac{dy}{dx} \quad \therefore \frac{dy}{dx} = \frac{f'(x)}{2y - 1}$$

(ii) If $y = f(x)^{f(x)^{f(x)^{\dots \infty}}}$ then $y = f(x)^y$.

$$\therefore \log y = y \log [f(x)]$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{y' \cdot f'(x)}{f(x)} + \log f(x) \cdot \left(\frac{dy}{dx} \right)$$

$$\therefore \frac{dy}{dx} = \frac{y^2 f'(x)}{f(x)[1 - y \log f(x)]}$$

(iii) If $y = f(x) + \frac{1}{f(x) + \frac{1}{f(x) + \frac{1}{f(x)} \dots}}$ then $\frac{dy}{dx} = \frac{y f'(x)}{2y - f(x)}$

DIFFERENTIATION AND APPLICATION

- **Interpretation of the Derivative :** If $y = f(x)$ then, $m = f'(a)$ is the slope of the tangent line to $y = f(x)$ at $x = a$

- **Increasing/Decreasing :**

- (i) If $f'(x) > 0$ for all x in an interval I then $f(x)$ is increasing on the interval I .
- (ii) If $f'(x) < 0$ for all x in an interval I then $f(x)$ is decreasing on the interval I .
- (iii) If $f'(x) = 0$ for all x in an interval I then $f(x)$ is constant on the interval I .

- **Test of Local Maxima and Minima –**

First Derivative Test – Let f be a differentiable function defined on an open interval I and $c \in I$ be any point. f has a local maxima or a local minima at $x = c$, $f'(c) = 0$.

Put $\frac{dy}{dx} = 0$ and solve this equation for x . Let c_1, c_2, \dots, c_n be the roots of this.

If $\frac{dy}{dx}$ changes sign from +ve to -ve as x increases through c_1 then the function attains a local max at $x = c_1$

If $\frac{dy}{dx}$ changes its sign from -ve to +ve as x increases through c_1 then the function attains a local minimum at $x = c_1$

If $\frac{dy}{dx}$ does not change sign as x increases through c_1 then $x = c_1$ is neither a point of local max^m nor a point of local min^m. In this case x is a point of inflexion.

➤ **Rate of change of variable :**

The value of $\frac{dy}{dx}$ at $x = x_0$ i.e. $\left(\frac{dy}{dx}\right)_{x=x_0}$ represents the rate of change of y with respect to x at $x = x_0$

If $x = \phi(t)$ and $y = \psi(t)$, then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, provided that $\frac{dx}{dt} \neq 0$

Thus, the rate of change of y with respect to x can be calculated by using the rate of change of y and that of x each with respect to t .

➤ Length of Sub-tangent = $\left| y \frac{dx}{dy} \right|$; Sub-normal = $\left| y \frac{dy}{dx} \right|$;

$$\text{Length of tangent} = \left| y \sqrt{1 + \left(\frac{dx}{dy} \right)^2} \right|$$

$$\text{Length of normal} = \left| y \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \right|$$

➤ **Equations of tangent and normal :** The equation of the tangent at $P(x_1, y_1)$ to the curve $y = f(x)$ is $y - y_1$

$= \left(\frac{dy}{dx} \right)_P (x - x_1)$. The equation of the normal at $P(x_1, y_1)$

to the curve $y = f(x)$ is $y - y_1 = -\frac{1}{\left(\frac{dy}{dx} \right)_P} (x - x_1)$

INTEGRAL CALCULUS

➤ **Two standard forms of integral :**

$$\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$$

$$\Rightarrow \int e^x [f(x) + f'(x)] dx = \int e^x f(x) dx + \int e^x f'(x) dx$$

$$= e^x f(x) - \int e^x f'(x) dx + \int e^x f'(x) dx$$

(on integrating by parts) $= e^x f(x) + c$

➤ Table shows the partial fractions corresponding to different type of rational functions :

S. No.	Form of rational function	Form of partial fraction
1.	$\frac{px + q}{(x - a)(x - b)}$	$\frac{A}{(x - a)} + \frac{B}{(x - b)}$
2.	$\frac{px^2 + qx + r}{(x - a)^2(x - b)}$	$\frac{A}{(x - a)} + \frac{B}{(x - a)^2} + \frac{C}{(x - b)}$
3.	$\frac{px^2 + qx + r}{(x - a)(x^2 + bx + c)}$	$\frac{A}{(x - a)} + \frac{Bx + C}{x^2 + bx + c}$

➤ **Leibnitz rule :** $\frac{d}{dx} \int_{f(x)}^{g(x)} F(t) dt = g'(x)F(g(x)) - f'(x)F(f(x))$

➤ If a series can be put in the form

$$\frac{1}{n} \sum_{r=0}^{n-1} f\left(\frac{r}{n}\right) \text{ or } \frac{1}{n} \sum_{r=1}^n f\left(\frac{r}{n}\right), \text{ then its limit as } n \rightarrow \infty$$

$$\text{is } \int_0^1 f(x) dx$$

➤ **Area between curves :**

$$y = f(x) \Rightarrow A = \int_a^b [\text{upper function}] - [\text{lower function}] dx$$

$$\text{and } x = f(y) \Rightarrow A = \int_c^d [\text{right function}] - [\text{left function}] dy$$

If the curves intersect then the area of each portion must be found individually.

➤ **Symmetrical area :** If the curve is symmetrical about a coordinate axis (or a line or origin), then we find the area of one symmetrical portion and multiply it by the number of symmetrical portion to get the required area.

PROBABILITY

➤ **Probability of an event:** For a finite sample space with equally likely outcomes Probability of an event is

$$P(A) = \frac{n(A)}{n(S)}, \text{ where } n(A) = \text{number of elements in the set}$$

$A, n(S) = \text{number of elements in the set } S.$

➤ **Theorem of total probability :** Let $\{E_1, E_2, \dots, E_n\}$ be a partition of a sample space and suppose that each of E_1, E_2, \dots, E_n has nonzero probability. Let A be any event associated with S , then

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + \dots + P(E_n)P(A|E_n)$$

➤ **Bayes' theorem:** If E_1, E_2, \dots, E_n are events which constitute a partition of sample space S , i.e. E_1, E_2, \dots, E_n are pairwise disjoint and $E_1 \cup E_2 \cup \dots \cup E_n = S$ and A be any event with nonzero probability, then

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{j=1}^n P(E_j)P(A|E_j)}$$

➤ Let X be a random variable whose possible values $x_1, x_2, x_3, \dots, x_n$ occur with probabilities $p_1, p_2, p_3, \dots, p_n$ respectively.

The mean of X , denoted by μ , is the number $\sum_{i=1}^n x_i p_i$

The mean of a random variable X is also called the expectation of X , denoted by $E(X)$.

➤ Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :

(a) There should be a finite number of trials. (b) The trials should be independent. (c) Each trial has exactly two outcomes : success or failure. (d) The probability of success remains the same in each trial.

For Binomial distribution $B(n, p)$,

$$P(X = x) = {}^nC_x q^{n-x} p^x, x = 0, 1, \dots, n \quad (q = 1 - p)$$

MATRICES

➤ **Properties of Transpose**

(i) $(A^T)^T = A$

(ii) $(A \pm B)^T = A^T \pm B^T$

(iii) $(AB)^T = B^T A^T$ (iv) $(kA)^T = k(A)^T$

(v) $I^T = I$ (vi) $\text{tr}(A) = \text{tr}(A)^T$

(vii) $(A_1 A_2 A_3 \dots A_{n-1} A_n)^T = A_n^T A_{n-1}^T \dots A_3^T A_2^T A_1^T$

➤ **Symmetric Matrix** : A square matrix $A = [a_{ij}]$ is called symmetric matrix if

$$a_{ij} = a_{ji} \text{ for all } i, j \text{ or } A^T = A$$

➤ **Skew-Symmetric Matrix** : A square matrix $A = [a_{ij}]$ is called skew-symmetric matrix if

$$a_{ij} = -a_{ji} \text{ for all } i, j \text{ or } A^T = -A$$

Also every square matrix A can be uniquely expressed as a sum of a symmetric and skew-symmetric matrix.

➤ **Differentiation of a matrix** : If $A = \begin{pmatrix} ef(x) & g(x) \\ eh(x) & l(x) \end{pmatrix}$ then

$$\frac{dA}{dx} = \begin{pmatrix} ef'(x) & g'(x) \\ eh'(x) & l'(x) \end{pmatrix}$$

is a differentiation of Matrix A

DETERMINANTS

➤ **Properties of adjoint matrix** : If A, B are square matrices of order n and I_n is corresponding unit matrix, then

(i) $A(\text{adj } A) = |A| I_n = (\text{adj } A)A$

(ii) $|\text{adj } A| = |A|^{n-1}$

(Thus $A(\text{adj } A)$ is always a scalar matrix)

(iii) $\text{adj}(\text{adj } A) = |A|^{n-2} A$

(iv) $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

(v) $\text{adj}(A^T) = (\text{adj } A)^T$

(vi) $\text{adj}(AB) = (\text{adj } B)(\text{adj } A)$

(vii) $\text{adj}(A^m) = (\text{adj } A)^m, m \in \mathbb{N}$

(viii) $\text{adj}(kA) = k^{n-1}(\text{adj } A), k \in \mathbb{R}$

(ix) $\text{adj}(I_n) = I_n$

➤ **Properties of Inverse Matrix** : Let A and B are two invertible matrices of the same order, then

(i) $(A^T)^{-1} = (A^{-1})^T$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

(iii) $(A^k)^{-1} = (A^{-1})^k, k \in \mathbb{N}$

(iv) $\text{adj}(A^{-1}) = (\text{adj } A)^{-1}$

(v) $(A^{-1})^{-1} = A$

(vi) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

(vii) If $A = \text{diag}(a_1, a_2, \dots, a_n)$, then

$$A^{-1} = \text{diag}(a_1^{-1}, a_2^{-1}, \dots, a_n^{-1})$$

(viii) A is symmetric matrix $\Rightarrow A^{-1}$ is symmetric matrix.

➤ **Rank of a Matrix** : A number r is said to be the rank of a $m \times n$ matrix A if

(a) Every square sub matrix of order $(r+1)$ or more is singular and (b) There exists at least one square submatrix of order r which is non-singular.

Thus, the rank of matrix is the order of the highest order non-singular sub matrix.

➤ Using Cramer's rule of determinant we get

$$\frac{x}{D_1} = \frac{y}{D_2} = \frac{z}{D_3} = \frac{1}{D} \text{ i.e. } x = \frac{D_1}{D}, y = \frac{D_2}{D}, z = \frac{D_3}{D}$$

Case-I : If $\Delta \neq 0$

$$\text{Then } x = \frac{D_1}{D}, y = \frac{D_2}{D}, z = \frac{D_3}{D}$$

\therefore The system is consistent and has unique solutions.

Case-II if $\Delta = 0$ and

(i) If at least one of $\Delta_1, \Delta_2, \Delta_3$ is not zero then the system of equations is inconsistent i.e. has no solution.

(ii) If $d_1 = d_2 = d_3 = 0$ or $\Delta_1, \Delta_2, \Delta_3$ are all zero then the system of equations has infinitely many solutions.

VECTOR ALGEBRA

➤ Given vectors $x_1 \vec{a} + y_1 \vec{b} + z_1 \vec{c}, x_2 \vec{a} + y_2 \vec{b} + z_2 \vec{c}, x_3 \vec{a} + y_3 \vec{b} + z_3 \vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors,

$$\text{will be coplanar if and only if } \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

➤ **Scalar triple product** :

(a) If $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}, \vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$ and $\vec{c} = c_1 \hat{i} + c_2 \hat{j} + c_3 \hat{k}$ then

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = [\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(b) $[a \ b \ c] = \text{volume of the parallelopiped whose coterminous edges are formed by } \vec{a}, \vec{b}, \vec{c}$

(c) $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if $[\vec{a} \ \vec{b} \ \vec{c}] = 0$

(d) Four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively are coplanar if and only if

$$[\vec{AB} \ \vec{AC} \ \vec{AD}] = 0 \text{ i.e. if and only if}$$

$$[\vec{b} - \vec{a} \ \vec{c} - \vec{a} \ \vec{d} - \vec{a}] = 0$$

(e) Volume of a tetrahedron with three coterminous edges

$$\vec{a}, \vec{b}, \vec{c} = \frac{1}{6} |[\vec{a} \ \vec{b} \ \vec{c}]|$$

(f) Volume of prism on a triangular base with three

$$\text{coterminous edges } \vec{a}, \vec{b}, \vec{c} = \frac{1}{2} |[\vec{a} \ \vec{b} \ \vec{c}]|$$

➤ **Lagrange's identity :**

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix} = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

➤ **Reciprocal system of vectors :** If $\vec{a}, \vec{b}, \vec{c}$ be any three non coplanar vectors so that

$[\vec{a} \vec{b} \vec{c}] \neq 0$ then the three vectors $\vec{a}', \vec{b}', \vec{c}'$ defined by the

$$\vec{a}' = \frac{\vec{b} \times \vec{c}}{[\vec{a} \vec{b} \vec{c}]}, \vec{b}' = \frac{\vec{c} \times \vec{a}}{[\vec{a} \vec{b} \vec{c}]}, \vec{c}' = \frac{\vec{a} \times \vec{b}}{[\vec{a} \vec{b} \vec{c}]}$$

are called the reciprocal system of vectors to the given vectors $\vec{a}, \vec{b}, \vec{c}$

STATISTICS

➤ **Relation between A.M., G.M. and H.M.**

$$A.M. \geq G.M. \geq H.M.$$

Equality sign holds only when all the observations in the series are same.

➤ **Relationship between mean, mode and median :**

(i) In symmetrical distribution

$$\text{Mean} = \text{Mode} = \text{Median}$$

(ii) In skew (moderately symmetrical) distribution

$$\text{Mode} = 3 \text{ median} - 2 \text{ mean}$$

➤ **Mean deviation for ungrouped data**

$$M.D.(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n}, \quad M.D.(M) = \frac{\sum |x_i - M|}{n}$$

➤ **Mean deviation for grouped data**

$$M.D.(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{N}, \quad M.D.(M) = \frac{\sum f_i |x_i - M|}{N},$$

where $N = \sum f_i$

➤ **Variance and standard deviation for ungrouped data**

$$\sigma^2 = \frac{1}{n} \sum (x_i - \bar{x})^2, \quad \sigma = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}$$

➤ **Variance and standard deviation of a discrete frequency distribution**

$$\sigma^2 = \frac{1}{n} \sum f_i (x_i - \bar{x})^2, \quad \sigma = \sqrt{\frac{1}{N} \sum f_i (x_i - \bar{x})^2}$$

➤ **Variance and standard deviation of a continuous frequency distribution**

$$\sigma^2 = \frac{1}{n} \sum f_i (x_i - \bar{x})^2, \quad \sigma = \sqrt{\frac{1}{N} \sum f_i x_i^2 - (\sum f_i x_i)^2}$$

➤ **Coefficient of variation (C.V.) = $\frac{\sigma}{\bar{x}} \times 100$, $\bar{x} \neq 0$**

For series with equal means, the series with lesser standard deviation is more consistent or less scattered.

DIFFERENTIAL EQUATIONS

➤ **Methods of solving a first order first degree differential equation :**

(a) **Differential equation of the form** $\frac{dy}{dx} = f(x)$

$$\frac{dy}{dx} = f(x) \Rightarrow dy = f(x) dx$$

Integrating both sides we obtain

$$\int dy = \int f(x) dx + c \quad \text{or } y = \int f(x) dx + c$$

(b) **Differential equation of the form** $\frac{dy}{dx} = f(x) g(y)$

$$\frac{dy}{dx} = f(x) g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x) dx + c$$

(c) **Differential equation of the form of** $\frac{dy}{dx} = f(ax + by + c)$:

To solve this type of differential equations, we put

$$ax + by + c = v \quad \text{and} \quad \frac{dy}{dx} = \frac{1}{b} \left(\frac{dv}{dx} - a \right)$$

$$\therefore \frac{dv}{a + b f(v)} = dx$$

$$\text{So solution is by integrating } \int \frac{dv}{a + b f(v)} = \int dx$$

(d) **Differential Equation of homogeneous type :**

An equation in x and y is said to be homogeneous if it

can be put in the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ where $f(x, y)$ and $g(x, y)$ are both homogeneous functions of the same degree in x & y .

So to solve the homogeneous differential equation

$$\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}, \quad \text{substitute } y = vx \quad \text{and so} \quad \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\text{Thus } v + x \frac{dv}{dx} = f(v) \Rightarrow \frac{dx}{x} = \frac{dv}{f(v) - v}$$

$$\text{Therefore solution is } \int \frac{dx}{x} = \int \frac{dv}{f(v) - v} + c$$

➤ **Linear differential equations :**

$$\frac{dy}{dx} + Py = Q \quad \dots\dots (1)$$

Where P and Q are either constants or functions of x .

Multiplying both sides of (1) by $e^{\int P dx}$, we get

$$e^{\int P dx} \left(\frac{dy}{dx} + Py \right) = Q e^{\int P dx}$$

On integrating both sides with respect to x we get

$$y e^{\int P dx} = \int Q e^{\int P dx} + c$$

which is the required solution, where c is the constant and

$e^{\int P dx}$ is called the integration factor.

THE LIVING WORLD & TAXONOMY

- **Nomenclature** – It is the process of giving scientific names (not vernacular or local names) to the organisms.
- **Systematics** – This includes the identification, nomenclature and classification of organisms based on various parameters.
- In **binomial nomenclature** each scientific name has 2-components,
 - i. Generic name (Genus),
 - ii. Specific name/epithet (Species)
- Various levels of classification are –
Kingdom → Phylum (animals)/ Division (plants) → Class → Order → Family → Genus → Species.
- Species is the basic unit of classification.
- **Artificial system of Classification** - All taxonomists, from Aristotle to Linnaeus, classified organisms on the basis of external observable (morphological) characters like floral structure (number of stamens), root modification, leaf venation etc. In this system, no weightage was given to natural and phylogenetic relationship.
- **Natural System of Classification** - It uses more number of characters and is based upon natural affinities using homology and comparative study. Bentham - Hooker used this system of classification for angiosperms.
- **Phylogenetic Classification (Cladistics)** - This system of classification is based upon evolutionary relationship and uses morphological characters, origin and evolution of the different organisms.

BIOLOGICAL CLASSIFICATION

- Characteristics of five kingdom system was proposed by R.H. Whittaker (1969). Accordingly, a separate kingdom has been created for Fungi. Thus, these are – Monera, Protista, Fungi, Plantae and Animalia.
- Bacteria are prokaryotes. They lack nucleus and other cell organelles of complex cells and have prokaryotic ribosome (70 S). Like plants, they possess cell wall.
- The cyanobacteria are the largest and most diverse group of photosynthetic bacteria, *e.g.*, *Nostoc*. Heterocyst is specialised to perform nitrogen fixation.
- Mycoplasma are the smallest known aerobic prokaryotes without cell wall.
- Haeckel (1886) created the kingdom protista to include all unicellular eukaryotic microorganisms. *E.g.* Euglenoids are photosynthetic in the presence of sunlight, when deprived of sunlight they behave like heterotrophs by predating on other smaller organisms.

- Fungi are achlorophyllous, heterotrophic, gametophytic, haploid, multicellular, eukaryotic nucleated, spore producing thallophytes which are surrounded by cell wall of chitin (fungus cellulose). Study of fungi is called Mycology.
- Viruses are obligate parasites, *i.e.*, can live inside living host only. They have either RNA or DNA. Bacterial viruses or bacteriophages (viruses that infect the bacteria) are usually double stranded DNA viruses.
- Viroids contain only very low molecular weight RNA and protein coat cause persistent infections, *i.e.*, never recovered.
- In lichens, there are 2 components; *i.e.*, algal partner called phycobiont and fungal partner called mycobiont.

PLANT AND ANIMAL KINGDOMS

- Bryophytes are terrestrial plants but they require water for dehiscence of antheridia, liberation and swimming of antherozoids, fertilization of egg, opening of archegonial neck and entry of sperms into the archegonium. Because of such peculiar habitat they have been most appropriately called the **amphibians of the plant kingdom**.
- Evolutionarily, Pteridophytes are the first terrestrial plants to possess vascular tissues – xylem and phloem. So known as vascular cryptogams. They are flowerless and seedless plants.
- Gymnosperms are plants which bear naked seeds *i.e.*, the ovules and the seeds that develop from these ovules after fertilization are not enclosed in fruit wall. In most gymnosperms, secondary growth occurs and annual rings are distinct. The xylem is without vessels (except in *Gnetales*) and phloem lacks companion cells.
- In angiosperms, the male sex organs in a flower is the stamen. Each stamen consists of a slender filament with an anther at the tip. It produces pollen grains.
- The female sex organs in a flower is the pistil or the carpel. Pistil consists of an ovary enclosing one to many ovules. Within ovules are present highly reduced female gametophytes termed embryo sacs. The embryo-sac formation is preceded by meiosis. Hence, each of the cells of an embryo-sac is haploid. Each embryo-sac has a three-celled egg apparatus – one egg cell and two synergids, three antipodal cells and two polar nuclei. The polar nuclei eventually fuse to produce a diploid secondary nucleus.
- Pollen grain shed at 2-3 celled stage after dispersal from the anthers, are carried by wind or various other agencies to the stigma of a pistil. This is termed as **pollination**.
- The pollen tubes enter the embryo-sac where two male gametes are discharged. One of the male gametes fuses with the egg cell to form a zygote (syngamy). The other male gamete fuses with the diploid secondary nucleus to produce the triploid primary endosperm nucleus (PEN) known as triple fusion. Because of the involvement of two fusions, this event is termed as double fertilisation, and it is unique to angiosperms.

The zygote develops into an embryo (with one or two cotyledons) and the PEN develops into endosperm which provides nourishment to the developing embryo.

A fruit is a ripened ovary.

Life cycles in different plant groups differ in following manner:

Haplontic life cycle: There is no true alternation of generation as sporophytic generation is represented only by one celled zygote. There is no free living sporophyte.

Diplontic life cycle : The sporophyte is independent photosynthetic phase of the plant. Gametophytic phase is represented by haploid gametes only.

Haplo-diplontic life cycle : This is an intermediate condition where both phases are multicellular and often free living.

Animalia – They are eukaryotic, multicellular and heterotrophic organisms that do not have cell wall. On the basis of extent and type of body design, they are classified as–

- (i) Porifera – E.g. *Spongilla*, *Sycon*.
- (ii) Coelenterata – E.g. Jelly fish, Sea anemone
- (iii) Platyhelminthes – E.g. *Planaria*.
- (v) Annelida – E.g. Earthworms.
- (vi) Arthropoda – E.g. Prawns.
- (vii) Mollusca – E.g. Snails.
- (viii) Echinodermata – E.g. Starfish.

Vertebrates are divided into 5 classes :

- (i) Pisces – Include fishes which are aquatic, E.g. Shark, Rohu.
- (ii) Amphibia – Animals are found both in water and on land, E.g. Toads, Frogs, Salamander.
- (iii) Reptilia – They are cold blooded animals with scales and breathe through lungs. E.g. Snakes, Turtles, Crocodiles etc.
- (iv) Aves – They have feathers and forelimbs are modified for flight. E.g. Pigeon, Sparrow, Ostrich.
- (v) Mammalia – They are warm blooded animals with 4 – chambered heart. They have mammary glands for milk production.

MORPHOLOGY OF FLOWERING PLANTS

A typical root can be differentiated into five distinct regions namely region of root cap, meristematic region, region of elongation, region of root hair and mature region.

Roots are divided into two types:

- (i) Tap root : It forms lateral branches or secondary roots which are further branched to form tertiary roots.
- (ii) Adventitious roots : These roots develop from any part of the plant instead of radicle.

Shoot system is an aerial system, usually above the soil and originates from the plumule. It consists of stem, branches, leaves, flowers, fruits and seeds.

The various types of underground modifications of stem are: Rhizome : *Zingiber* (ginger). Corm : Saffron (*Crocus*) Tuber : *Solanum tuberosum* (potato). Bulb : (onion)

An inflorescence is the mode of arrangement of flowers on peduncle or mother axis.

The main axis of *racemose* inflorescence has indefinite growth because there is no terminal flower. In cymose, the growth of the main axis is definite because the growing point of peduncle is used up in the formation of a flower.

Symbols For Floral Formula

Br	Bracteate
	Actinomorphic
% or †	Zygomorphic
♂	Staminate (male)
♀	Pistillate (female)
	Calyx (K)
	Corolla (C)
	Perianth (P)
	Androecium (A)
	Gynoecium (G)
G(2)	Bicarpellary, syncarpous, inferior
G(2)	Bicarpellary, syncarpous, superior

PLANT TISSUES

A group of structurally similar or dissimilar cells that perform a common function and have a common origin is called a tissue.

Cells which are capable of active cell division are called meristematic cells.

On the basis of position, the meristematic tissues can be divided into the following three types:

- Apical meristem
- Intercalary meristem
- Lateral meristem

Intercalary meristem is present at the base of internodes, *e.g.* in grasses (Gramineae) or at the base of leaves, *e.g.*, in *Pinus* or at the base of nodes, *e.g.*, mint or *Mentha* (Labiatae).

Simple Permanent Tissue

1. Parenchyma : It helps in storage of food, conduction of substances, provides turgidity to softer parts of plants.
2. Collenchyma : It is living mechanical tissue, found beneath the epidermis (*i.e.*, hypodermis) of herbaceous dicot stem.
3. Sclerenchyma : These are dead, mechanical tissue and act as skeleton in plants.

Complex Permanent Tissues

1. Xylem is mainly responsible for conduction of water and minerals from the roots to the top of plants (unidirectional). It also provides mechanical support to the plant. Xylem is made of 4 types of cells *i.e.*, tracheids, tracheae (vessels), xylem fibres and xylem parenchyma.
2. Phloem mainly carries food. It is a complex tissue made up of 4 kinds of cells in angiosperms. These are -sieve elements, companion cells, phloem fibres, phloem parenchyma.

ANIMAL TISSUES

Epithelial tissue – It forms protective covering in the animal body, covers organs and separates different body systems. Different types of epithelial tissues on the basis of functions are–

- (i) Simple squamous epithelium – Oesophagus, lining of mouth are made of it.
- (ii) Stratified squamous epithelium– In the skin epithelial cells are arranged in many layers to prevent wear and tear.
- (iii) Columnar epithelium – It is present in the inner lining of intestine, pillar like tall cells.

(iv) Cuboidal epithelium – It forms lining of kidney tubules and ducts of salivary glands.

(v) Glandular epithelium – In glands.

► **Connective Tissue** – The cells are loosely packed and embedded in intercellular matrix.

(i) Areolar tissue – It fills space inside organs, helps in repair of tissues.

(ii) Adipose tissue – It is fat storing, found below the skin and between internal organs.

(iii) Bone – Bone cells lie embedded in hard matrix composed of calcium and phosphorus.

(iv) Ligament – connects two bones.

(v) Tendons – connect muscles to bones.

(vi) Cartilage – It is present in nose, ear, trachea, larynx.

(vii) Blood – It is a fluid connective tissue. Fluid matrix is called plasma that contains red blood cells (RBCs), white blood cells (WBCs) and platelets unit.

► **Muscular tissue** – It consists of elongated cells called muscle fibres. They are responsible for movement in our body. The three types of muscles are –

(i) Striated muscles – E.g. muscles of hands and legs.

(ii) Smooth muscles – E.g. muscles found in iris of eye, ureters, bronchi of lungs, alimentary canal.

(iii) Cardiac muscles – E.g. the muscles of heart.

► **Nervous tissue** – They are found in brain, spinal cord and nerves. They transmit the stimulus. A neuron consists of a cell body with a nucleus and cytoplasm. It has a long part called axon and many short, branched parts called dendrites. Many nerve fibres together form a nerve.

CELL : THE UNIT OF LIFE

► Cell theory was proposed by Schleiden and Schwann.

► All living organisms (animals, plants and microbes) are made up of cells and cell products *i.e.*, cell is the structural unit of life.

► S.J. Singer and G. Nicolson in 1972 proposed the most accepted model of membrane structure only after the advent of electron microscope in 1950. According to this model, the lipid bilayer is a semifluid in which the proteins are dispersed to give mosaic appearance to the whole membrane.

► Plasmodesmata are characteristic of multicellular plants that enables the solutes to move considerable distances through the pits without crossing differentially permeable membranes.

► Endoplasmic reticulum is also called as ‘endoskeleton of the cell’ or ‘endomembranous system of the cell’. The endoplasmic reticulum is bounded by a single unit membrane. Smooth or agranular ER : They do not have attached ribosomes on their surface. The SER is well developed in those cells which are actively concerned with the synthesis and secretion of steroid hormone.

Rough or granular ER : They bear ribosomes on their surface. The cells which are active in protein synthesis have RER in abundance.

► Golgi apparatus or Golgi complex is a stack of flattened, membrane bound, parallel arranged organelles that occur in the association of endoplasmic reticulum in the cytoplasmic matrix. It principally performs the function of packaging materials to be delivered either to the intra-cellular targets or secreted outside the cell.

► Lysosomes are originated by Golgi complex and contain many hydrolytic enzymes (amylases, lipases, proteases, carbohydrases) optically active at the acidic pH. They are popularly called “suicide bags”.

► The vacuole is bound by a single membrane called tonoplast. It contains many dissolved solutes such as organic acids, soluble carbohydrates, soluble nitrogenous compounds as nitrates, enzymes, tannins, chlorides, phosphates, amino acids, alkaloids and anthocyanin pigments.

► Cilia and flagella are contractile filamentous or hair-like outgrowths of the cytoplasm, which help in cell movement, excretion, secretion and transportation.

► Centrosome is an organelle usually containing two cylindrical structures called centrioles. Centrioles duplicate before cell division so that dividing cell has a pair of centrioles at each pole.

► Peroxisomes are the smallest membrane bound cell organelles. These organelles were named peroxisomes because these are the sites of formation of hydrogen peroxide (H_2O_2).

► A typical nucleus consists of four structures: nuclear membrane, nucleoplasm, chromatin and the nucleolus.

► Nucleolus is site of active ribosomal RNA synthesis.

► In interphasic nucleus, the genetic material (DNA) is seen as the thread like structure called chromatin or chromatin fibres. During cell divisions (mitosis and meiosis), chromatin fibres become condensed to form thick structures called chromosomes.

► Based on the position of the centromere, the chromosomes can be classified into four types :

- metacentric (middle centromere)
- sub-metacentric (centromere nearer to one end of the chromosome)
- acrocentric (centromere situated close to its end)
- telocentric (terminal centromere).

► Plastids are found in plants and few protists (*Euglena*). They may be colourless (leucoplasts) or coloured (chromoplasts and chloroplasts). Each chloroplast has two parts *i.e.*, grana and stroma.

► Stroma : It is the enzymatic part of chloroplast. Stroma contains proteins, DNA, ribosomes and fewer amounts of enzymes. Dark reaction of photosynthesis occurs in this fraction.

► Grana : In chloroplast, several membranes are arranged in the form of stacks of coins. These thylakoids are stacked one above the other to form grana. Due to presence of chlorophyll in granum lamella, the light reaction of photosynthesis takes place in granum.

► Mitochondria are referred as “powerhouse” of the cell as they produce 95% of ATP. This energy is produced during the break down of food molecules which involve glycolysis, oxidative decarboxylation and oxidative phosphorylation (krebs cycle and respiratory chain).

BIOMOLECULES

► Lactose - It is milk sugar.

► Sucrose – It is non-reducing sugar as it does not have free aldehyde or ketose groups.

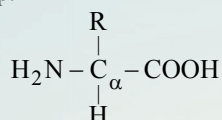
► Glycogen - It is present in animals (also called animal starch). It is a branched chain compd. and has about 30 α -glucose units.

► Starch - It is present in plants. The natural starch contains a mixture of amylose (10-20%) and amylopectin (80-90%).

➤ Cellulose - It is a fibrous polysaccharide and forms cell wall in plants.

➤ Proteins contain C, H, O, N. Some contain S (Sulphur) and P (Phosphorus) also. The structural unit of protein is amino acid. In **primary structure**, only Peptide bonds are present between amino acids. The first (or left) amino acid is called N-terminal ($-\text{NH}_2$) amino acid, and the last (or right) amino acid is called C-terminal ($-\text{COOH}$ gp.) amino acid. In **secondary structure**, besides peptide bonds, Hydrogen bonds are also present in polypeptide chain. Such proteins may have helical or pleated structures. The **tertiary structure** may result from further folding and coiling, and may be stabilized by S-S (disulphide) bond, hydrophobic bonds and ionic bonds. In **quaternary structure**, more than one polypeptide chains are involved to form a large multiunit protein. e.g. Haemoglobin.

➤ All amino acids contain atleast 1-Amino group and 1-carboxylic group.



➤ The nucleic acid on hydrolysis yields 1-Pentose Sugar, 2-types of heterocyclic nitrogenous bases (Purines and Pyrimidines) and phosphoric acid.

Nucleic Acid	Purines	Pyrimidines
DNA	Adenine and Guanine	Cytosine and Thymine
RNA	Adenine and Guanine	Cytosine and Uracil

➤ ATP (Adenosine Triphosphate) is also a nucleotide. It contains 1-Adenine base, 1-Ribose sugar and 3-phosphate bonds. It is energy-rich compound, and is also called as 'energy currency'.

➤ Edwin Chargaff reported that net amount of adenine was equal to thymine ($A = T$) and amount of guanine was equal to cytosine ($G = C$). This means that total number of purines is equal to the total number of pyrimidines ($A + G = T + C$).

➤ **Double Helical Structure of DNA** : To explain base equivalence ($A / T, G / C$) and other properties of DNA, Watson and Crick (1953), based on X-ray diffraction studies, proposed double helical structure of DNA. The two strands of helix are anti-parallel, means $5' \rightarrow 3'$ Phosphodiester bonds (Sugar-phosphate groups) are oriented in opposite direction in 2-strands, there are 2 hydrogen bonds ($A = T$) and in between C & G there are 3 hydrogen bonds ($C \equiv G$).

➤ **RIBONUCLEIC ACID (RNA)** : It has single helical structure and is mainly of 3-types.

- m-RNA (Messenger RNA)
- r-RNA (Ribosomal RNA)
- t-RNA (Transfer RNA)

The enzymes are tertiary proteins. The energy required for a chemical reaction to proceed is called Activation energy. Higher is the affinity of an enzyme for a substrate the lower is its K_m value,

$$\text{i.e. } K_m \text{ value} \propto \frac{1}{\text{affinity}}$$

➤ The co-factor can be inorganic or organic in nature. The organic factor, if permanently attached to the enzymes, is called prosthetic group and if temporarily attached (only during reaction), is called co-enzyme. Most of the co-enzymes are derivatives of vitamin B & C.

CELL CYCLE AND CELL DIVISION

➤ The interphase is the longest phase of the mitotic cycle, lasts more than 95% of the duration of cell cycle. It has three sub-phases: G_1 , S and G_2 which occurs in this order followed by M phase.

➤ **Mitosis** : It is also called as equational division as the number of chromosomes in the parent and progeny cells is the same. So mitosis involves increase in number of cells without any change in genetic constitution. It is usually divided into several stages or phases, viz. prophase, metaphase, anaphase, telophase and cytokinesis.

➤ **Meiosis** : It ensures the production of haploid phase in the life cycle of sexually reproducing organisms whereas fertilisation restores the diploid phase. Thus, meiosis is required to run the reproductive cycle of eukaryotes.

TRANSPORT IN PLANTS

➤ Diffusion — It is the movement of a substance from a region of higher concentration to a region of lower concentration.

➤ Osmosis — It is the movement of water through a selectively permeable membrane from a region of high water concentration to a region of low water concentration.

➤ The difference between free energy of water in a system & free energy of pure water at atmospheric pressure is also called water potential.

➤ The shrinkage of the protoplast of a living cell from its cell wall due to exosmosis under the influence of a hypertonic solution is called plasmolysis.

➤ The process of absorption of water by hydrophilic surfaces of a substance without forming a solution is called imbibition. Examples of imbibition are adsorption of water by seeds, dry wood, starch, cellulose, agar, gelatin, gum etc.

MINERAL NUTRITION

➤ Technique of growing plants in a nutrient solution is known as hydroponics or soilless growth. By this method, essential elements were identified and their deficiency symptoms discovered.

➤ Essential mineral elements are those elements which are required by plants for normal growth and development and without which plants cannot complete their life cycle.

E.g., C, H, O, N, P, K, S, Mg, Ca, Fe, B, Mn, Cu, Zn, Mo, Cl. Elements which are present in a plant body and are not so required by plants are called non-essential elements. e.g., Na, Si, Al, Se, Sr, V.

➤ Plant can not absorb nitrogen in molecular form. It is absorbed by plants in nitrate NO_3^- and ammonium NH_4^+ form.

➤ Conversion of gaseous nitrogen into nitrogenous compound by living organism like bacteria, cyanobacteria is called biological nitrogen fixation.

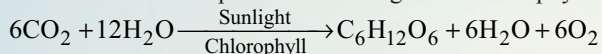
➤ The most prominent among them is the legume-bacteria relationship. Species of rod-shaped *Rhizobium* has such relationship with the roots of several legumes such as alfalfa, sweet clover, sweet pea, lentils, garden pea, broad bean, clover beans, etc. Nodules act as the site for N_2 fixation. It contains leghaemoglobin (a pink pigment) and enzyme nitrogenase (Mo-Fe protein).

PLANT AND ANIMAL PHYSIOLOGY

Nutrition is a process by which an organism obtains its food.

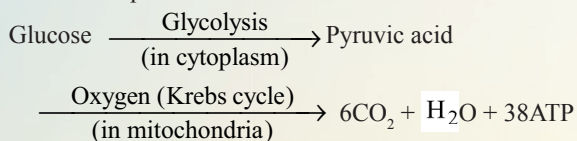
- Autotrophic Nutrition : synthesising food from the raw materials like carbon dioxide, water and a few minerals. e.g., green plants, Euglena.
- Heterotrophic Nutrition : Derives its nutrition from other organisms. e.g., all animals, most bacteria and fungi.
- Parasitic Nutrition : Live on or inside the body of the host and obtain their food. e.g., Tapeworm, Cuscuta (amarbel).
- Saprophytic Nutrition : Animals depend on dead decaying organic matters. e.g., fungi, bacteria.
- Holozoic Nutrition: The complex organic food material is taken into its body by the process of ingestion, e.g., man, cat, dog, fish, *Amoeba*, etc.

Photosynthesis is the process by which autotrophic chlorophyll containing organisms manufacture their own energy sources (simple sugars) from intracellular chemical reaction of carbon dioxide and water in presence of sunlight and chlorophyll.

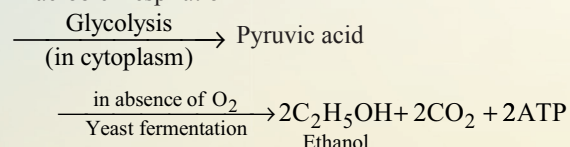


Respiration is a complex process which includes breathing i.e. exchange of O_2 and CO_2 and oxidation of food to release energy.

(a) Aerobic Respiration



(b) Anaerobic Respiration



In human beings, respiratory pigment is haemoglobin which remains present in RBC. CO_2 is more soluble in water than oxygen and hence is mostly transported in the dissolved form in blood. Due to difference in concentration of gases, exchange takes place between tissues and blood capillaries.

Transportation is a process in which substances are absorbed in one part and move to the other parts of the body.

Arteries are thick-walled and transport blood from heart to the various parts of the body. Veins are thin walled and carry blood from various organs to the heart. Heart is situated in the thoracic cavity between two lungs. Heart is made up of cardiac muscles which works continuously. Human heart has four chambers i.e. two auricles and two ventricles. Normal heart rate is 72/minute.

The removal of unwanted waste materials from the body is called **excretion**. Excretory system of human beings consists of a pair of kidneys, a pair of ureters, urinary bladder, urethra.

Improper functioning of the kidneys may lead to accumulation of water in the body called oedema.

An artificial kidney is a device to remove nitrogenous waste products from the blood through dialysis.

Oxygen is waste product of photosynthesis in plants. Many waste products are stored as resins and gums in plants.

Reproduction in Human Beings

- The sex organ in males are testes and ova in females.
- Male reproductive organ consist of a pair of testes, vas deferens, a pair of epididymis, a pair of ejaculatory duct, urethra, pairs of accessory gland. Leydig cells secrete male sex hormone i.e., testosterone which is concerned with the production of male sexual characters.
- Female reproductive part consist of a pair of ovaries, a pair of fallopian tube, uterus, vagina, external genitalia, mammary glands and accessory glands.
- If sperms are present, fertilization of ovum takes place in the upper end of the fallopian tube.
- Fertilization process occurs in fallopian tube. In this process, zygote is formed. Umbilical cord is produced which is attached to foetus. During this process, two hormones are produced which are estrogen and progesterone. Progesterone stops menstruation and prevents ovulation.

Human Brain :

- Brain is inside the cavity called cranium. It is filled with cerebrospinal fluid. It is divided into three parts –
 - Fore brain (Cerebrum) is the main thinking part of the brain. All our thoughts, sensations, actions and movements are controlled by the cerebrum.
 - Midbrain consist of nerve cells, connects forebrain to the hind brain. It has reflex centres for eye movement and hearing response.
 - Hind brain (Cerebellum) is the second largest part of brain. The surface is of grey matter while deeper part is of white matter. Cerebellum maintains posture, regulates muscle tone.
- Medulla oblongata controls involuntary movement, acts as reflex centre for vomiting, coughing, sneezing, swallowing etc.
- Reflex Action : It is rapid, automatic, definite response to stimulus by an organ without involving brain for its initiation. The pathway which is followed by this is called reflex arc. E.g. Knee-jerk reflex, sneezing , yawning , blinking of eye.

MOVEMENT IN PLANTS

Tropism (Tropic Movements) : A growth movement of a plant part in response to an external stimulus in which the direction of stimulus determines the direction of response is called tropism.

- Nastic (Nastic movements) : It is not a directional movement of the plant part with respect to the stimulus.
- Thigmonasty : It is the non-directional movement of a plant part in response to the touch of an object. E.g.- *Mimosa pudica* (Chui-mui).
- Photonasty : The non-directional movement of a plant part (usually petals of flowers) in response to light is called photonasty.

HORMONES

Growth hormones are the natural growth substances which are produced in any part of the plant.

- Auxins : It stimulates growth, phototropism, geotropism 2, 4 - D is used to avoid pre-harvest fruit in oranges, apples, used as weedicide.
- Gibberellins : These can increase the height of plant, can induce parthenocarp, stimulate flowering.

- (iii) Cytokinins : Promote cell division, inhibit or delay ageing, organ formation.
- (iv) Ethylene : It's a gaseous plant hormone, used in artificial ripening of fruits, promote ageing in plants, breaks dormancy of several organs.
- (v) Absciscic Acid (ABA) : Also known as stress hormone. It is a growth inhibitor, inhibits the process of flowering, seed development.

► Hormones in Animals :

- (i) Hypothalamus : The hypothalamic hormones are also called releasing hormones : TSH, ACTH, FSH, LH.
- (ii) Pituitary gland : It has two lobes anterior (adenohypophysis) and posterior (neurohypophysis).
 - (a) Anterior lobe : Secretes TSH (Thyroid Stimulating hormone), ACTH (Adrenocorticotrophic hormone), GH (Growth hormone), FSH (Follicle stimulating hormone), LH (Luteinizing hormone), Prolactin.
 - (b) Intermediate lobe : production of melanin pigment.
 - (c) Posterior lobe :
 - (A) Oxytocin : Released during child birth, during breast feeding.
 - (B) Vasopressin : It's an antidiuretic hormone which controls the secretion of urine by kidney.
- (iii) Pineal : It secretes biogenic amine hormone called melatonin. It inhibits ovarian growth and ovulation.
- (iv) Thyroid : Thyroxine promotes the growth of body tissues. Calcitonin lowers blood calcium level.
- (v) Parathyroid : It increases blood calcium level from bone to blood.
- (vi) Pancreas : It is exocrine as well as endocrine. The endocrine part is called islets of Langerhans. It consists of α and β cells. β cells secrete hormone insulin which regulates blood sugar level. Excess of glucose starts appearing in urine and the disease is called diabetes.

HEREDITY AND EVOLUTION

► Mendel's laws of inheritance

The first study of inheritance was done by Gregor Mendel on garden pea (*Pisum sativum*). He used a number of contrasting characters like round / wrinkled seeds, tall/ short plants, white/ violet flowers and so on.

► Law of Dominance : Out of a pair of allelomorphic characters one is dominant (expressed) and the other is recessive/unexpressed. The benefit of this law is that recessive (harmful) characters are not expressed in hybrid and can exist for several generations.

► Law of Segregation : The factors for each character segregate during gametogenesis. As a result, each gamete receives only one factor for each character and hence is always pure.

► Law of Independent Assortment : The two factors of each trait assort at random and independent of the factors of other trait at the time of meiosis and get randomly as well as independently arranged in the offspring.

SEX DETERMINATION

All human chromosomes are not paired. 22 pairs are called autosomes. Women have a perfect pair of sex chromosomes XX. But men have a mismatched pair XY.

EVOLUTION

It is the sequence of gradual changes which takes place in the primitive organisms over millions of years in which new species are produced.

The evidences of evolution are :

- (i) Homologous organs : The organs which have same fundamental structure but different functions.
- (ii) Analogous organs : The organs which have similar functions but are different in their structural detail and origin. E.g. wings of insect and wings of bird. The organ which are present in reduced form and do not perform any function in the body but correspond to the fully developed functional organs of related animals called vestigial organs
- (iii) Fossils : Fossils are the remains of the past and the study of fossils is known as paleontology. Charles Robert Darwin (1809-1882) explained the evolutionary principle in his famous book "The origin of species". The theory proposed by him is popularly known as theory of natural selection.

Speciation

The process by which new species develop from the existing species is known as speciation.

- (i) Geographical isolation of a population caused by various types of barriers (such as mountain ranges, rivers and sea).
- (ii) Genetic drift caused by drastic changes in the frequencies of particular genes is by chance alone.
- (iii) Variations caused in individuals due to natural selection.

IMPROVEMENT IN FOOD RESOURCES

► Green revolution has increased food grain production while white revolution has increased production of milk.

► Based on the seasons of cultivation, crops are classified in two categories:

- (i) **Kharif Crops** : Crops grown in rainy season from July to October are kharif crops . Example: Paddy, soyabean, maize, cotton, and green gram etc.
- (ii) **Rabi Crops** : Crops grown in winter season from November to April are rabi crops. Example: Wheat, gram, peas, mustard and linseed etc.

► Hybridisation : Crossing between genetically dissimilar plants is hybridization. The crossing may be intervarietal (between different varieties), intergeneric (between different genera), interspecific (between different species of the same genus). The most common type of breeding is intervarietal.

► Manure : It is prepared by decomposition of animal excreta and plant waste. Plants like sunhemp or guar are grown and then mulched by ploughing them into the soil.

► Organic farming: It is a farming system using organic manures, recycled farm-wastes and use of bio-agents such as culture of blue-green algae in preparation of biofertilisers, neem leaves or turmeric specifically in grain-storage as bio-pesticides.

► Irrigation : Various irrigation systems are adopted to supply water to agricultural lands, these are – wells, canals, rivers, tanks etc. Rain water harvesting and watershed management increases the availability of water.

► Cropping Patterns

- (i) **Mixed cropping** : Growing two or more crops simultaneously on same land is mixed cropping.
- (ii) **Inter-cropping** : Growing two or more crops simultaneously on the same field in a definite pattern is inter cropping.

(iii) **Crop-rotation** : Growing of different crops on a piece of land in a pre-planned succession is crop rotation.

- Animal husbandry : It is the scientific management of animal livestock in various aspects such as feeding, breeding and disease control. The animal included are cattle, goat, poultry, sheep and fish.
- Cattle farming: Indian cattle belong to two different species *Bos indicus* (cows) and *Bos bubalis* (buffaloes).
- Poultry farming : They are farmed to produce layers for eggs and broilers for meat.
- Fish production: Fish provides animal protein for us.
 - Capture fishing obtained from natural resources.
 - Culture farming obtained by fish farming.
- Bee-keeping : It has become an agricultural enterprise for honey production. It is a low investment activity and produces wax also. Local varieties of bees used for honey production are – *Apis cerana indica* (Indian bee), *A. dorsata* (Rock bee), *A. florea* (little bee).

PRINCIPLES OF INHERITANCE AND VARIATION

- Incomplete dominance is the phenomenon where the dominant allele does not completely express itself. Example, In *Mirabilis jalapa* (four O' clock).
 - In codominance, both the alleles of a gene are equally dominant i.e. the dominant character is not able to suppress the recessive character & thus both the characters appear side by side in the F_1 hybrids. F_1 generation resembles both the parents. E.g., ABO blood group in humans.
 - Linkage is the phenomenon in which certain genes staying together inherit through generations without any change or separation. This is due to their location on the same chromosomes.
 - The rearrangement of linked genes due to crossing over is known as recombination.
 - The phenomenon that results in alteration of DNA sequence and consequently results in change of genotype and phenotype of an organism is called mutation.
- Mutagens are various chemical and physical factors that induce mutations, e.g., UV radiations, carcinogenic chemicals like nicotine, nitric oxide (NO).

MOLECULAR BASIS OF INHERITANCE

- Formation of a new DNA strand from an old DNA is called DNA replication or DNA duplication.
- Process of copying genetic information from DNA to RNA is called transcription.
- Transfer of genetic information from a polymer of nucleotides to a polymer of amino acids is called translation. This is accomplished with the help of genetic code which is a row of three consecutive nucleotides – coding for 20 amino acids.

HEALTH AND DISEASES

- **AIDS** : Acquired Immuno Deficiency Syndrome, was first recognized in USA in 1981. It is caused by HIV (Human immunodeficiency virus), a retro virus having 2-strands of single stranded RNA (RNAs), with reverse transcriptase enzyme.
- **Vaccination** : It is the inoculation/injection of weak or

attenuated antigens, or a toxin, or a protein, into the body. The introduction of antigens stimulates the production of antibodies and memory cells, which protect the body against that antigen/disease.

- The study of cancer is called 'Oncology'. Cancer is the unregulated and uncontrolled proliferation of cells, or the breakdown of regulatory mechanism that governs normal cell division.
- Immunity is the resistance against pathogens, foreign materials and cancer etc. It is of 2-types.
 - (a) Innate Immunity : This immunity is by birth, and develops by virtue of genes.
 - (b) Acquired Immunity : It can be acquired before birth (from mother through placenta) or after birth. There is an antigen-antibody reaction in this type of immunity.

Antigens : 'Antigen' is an acronym for antibody generating material.

Antibodies : These are pure proteins (γ -globulins). Since they participate in the immune system, they are also known as immunoglobulins (Ig).

Drugs : Drugs are chemicals that alter the functioning of the body.

- (i) Sedatives and tranquillizers - eg. Barbiturates (used in sleeping pills), Valium,
- (ii) Opiate narcotics (opioids) - eg. Opium, Morphine, Pathedine and Heroin etc.

BIOTECHNOLOGY

- Biotechnology is the application of techniques using live organisms to get desired product of human welfare. It includes recombinant DNA, gene cloning, gene therapy.
- rDNA technology is hybridization of DNA from different sources to achieve desired genotype and phenotype in an organism.
- Restriction endonucleases can break DNA at specific sites. They are appropriately called molecular scissors or biological scissors.
- Vectors are cloning vehicles required to transfer DNA of interest from one organism to another.
- Plasmids are extra-chromosomal, circular, double stranded autonomously replicating DNA sequence in a bacterial cell.
- Cosmids can be defined as the hybrid vectors derived from plasmids which contain cos site of lambda phage.
- *Bt* cotton is the first genetically modified crop of the country.

ORGANISMS AND POPULATIONS

- **Adaptations** are the morphological, physiological or behavioural attributes that enable the organisms to survive and reproduce in their habitat.
- Population is a group of individuals of a particular species, which can potentially interbreed and live in a well defined geographical area, and also share or compete for similar resources.
- **Birth or Natality rate** – It is the number of births per thousand in a population per year.
- **Death or Mortality rate** – It is the number of deaths occurring in a population of one thousand per year.
- The growth of population with time shows specific and predictable patterns. The 2 common patterns are

● **Exponential growth**

$$\frac{dN}{dt} = (b - d) \times N$$

If $(b - d) = r$, then $\frac{dN}{dt} = rN$

Here 'r' is called 'Intrinsic rate of natural increase' or Biotic potential (maximum capacity of reproduction), which indicates the impact of biotic and abiotic factors in population growth.

● **Logistic growth**

$$\frac{dN}{dt} = rN \left(\frac{K - N}{K} \right)$$

K = Nature's carrying capacity in that habitat
 $(K - N)/K$ or $1 - N/K$ = environmental resistance

► **Population Interactions**

S. No.	Name of interaction	Species A	Species B
1	Parasitism	+	–
2	Commensalism	+	0
3	Mutualism	+	+
4	Predation	+	–
5	Competition	–	–
6	Amensalism	–	0

ECOSYSTEM

- Ecosystem is the functional unit of nature where living organisms interact with each other and with their environment.
- Productivity refers to the rate of biomass production i.e. the rate at which sunlight is captured by producers for the synthesis of energy rich organic compounds.
- Primary productivity is the amount of biomass produced per unit area over a time period by plants during photosynthesis.
- Gross primary productivity (GPP) – It is the rate of production of biomass or accumulation of energy by green plants per unit area per unit time. GPP depends on the chlorophyll content.
- Net primary productivity = Gross primary productivity – Respiration losses. (or $GPP - R = NPP$)
- Food chain is the sequence of different organisms which are arranged in a way that energy of food components is passed from one type of organism to other organisms such that the organisms of one order or trophic level are the food of the organisms of next order or trophic level.
- Food web refers to a group of inter-related food chains in a particular community.
- Ecological succession is the successive development of different biotic communities at the same site. The communities develop one after another till the development of a community which is near equilibrium with the environmental conditions. This is called the climax community.

BIODIVERSITY AND ITS CONSERVATION

- Biodiversity means diversity or heterogeneity at all levels of biological organization, i.e., from macromolecules of the cells to the biomass.
- The important levels of biodiversity are
 - (i) Genetic diversity,
 - (ii) Species diversity
 - (iii) Ecological diversity
- Ecological diversity

It is the diversity at community level. It can be of 3 types

 - (i) Alpha (α) diversity : It is the diversity of organisms within the same community or habitat.
 - (ii) Beta (β) diversity : It is the diversity between communities or different habitats.
 - (iii) Gamma (γ) diversity : It is the diversity of organisms over the entire geographical area, covering several ecosystems or habitats and various trophic levels and food webs.
- **Biosphere reserves** – They represent natural biomes which contain unique biological communities.
- **National Parks** – They are reserved for the betterment of wild life, (both fauna and flora).
- **Sanctuaries** – In sanctuaries protection is given to fauna only. Activities like harvesting of timber, collection of forest products and private ownership rights are permitted so long as they do not interfere with the well being of the animals.

ENVIRONMENTAL ISSUES

- Pollution is an undesirable change in physical, chemical or biological characteristics of air, land, water or soil. The agents that bring about such an undesirable change are called pollutants.
- In a green house the glass panel lets the light (with long wavelength in, but does not allow heat (with short wavelength) to escape.
- The term 'Green House Effect' is derived from this phenomenon of green house. It is a naturally occurring phenomenon that is responsible for heating of earth's surface and atmosphere.
- The ozone (O_3) found in upper part of the atmosphere, i.e., Stratosphere, is good ozone, since, it acts as a shield for absorbing UV-radiations from sun.
- Ozone depletion is occurring widely in the stratosphere, the depletion is particularly marked over 'Antarctic region'; and this has resulted in the formation of a large area of thinned ozone layer called 'Ozone Hole'.
- Deforestation is the conversion of the forested area into non-forested area.