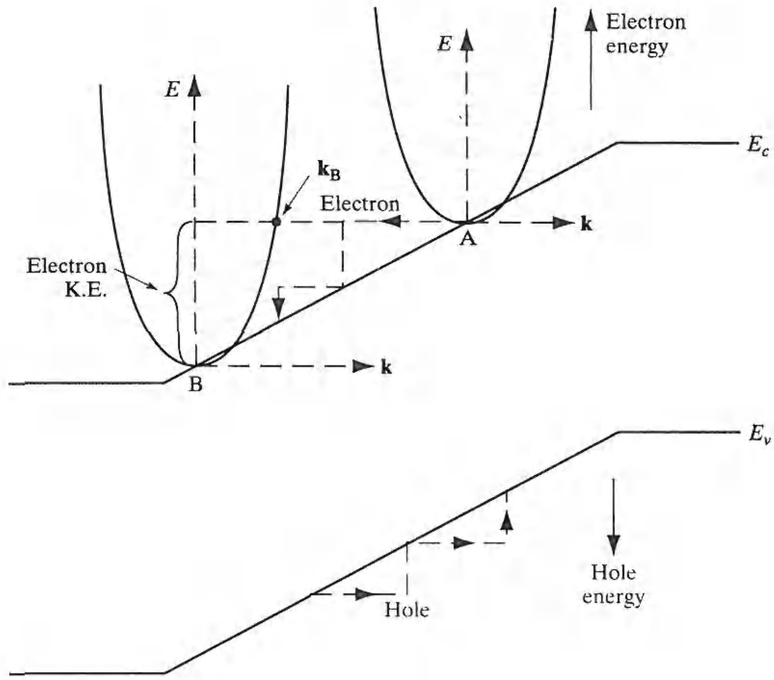


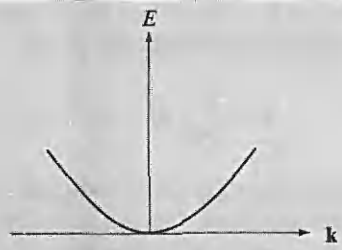
Figure 3–9 Superimposition of the $(E,\mathbf{k})$ band structure on the E-versus-position simplified band diagram for a semiconductor in an electric field. Electron energies increase going up, while hole energies increase going down. Similarly, electron and hole wave vectors point in opposite directions and these charge carriers move opposite to each other, as shown.

The electron then loses kinetic energy to heat by scattering mechanisms (discussed in Section 3.4.3) and returns to the bottom of the band at B. The slopes of the (E, x) band edges at different points in space reflect the local electric fields at those points. In practice, the electron may lose its kinetic energy in stages by a series of scattering events, as shown by the colored dashed lines.

3.2.2 Effective Mass

The electrons in a crystal are not completely free, but instead interact with the periodic potential of the lattice. As a result, their "wave-particle" motion cannot be expected to be the same as for electrons in free space. Thus, in applying the usual equations of electrodynamics to charge carriers in a solid, we must use altered values of particle mass. In doing so, we account for most of the influences of the lattice, so that the electrons and holes can be treated as "almost free" carriers in most computations. The calculation of effective mass must take into account the shape of the energy bands in three-dimensional **k**-space, taking appropriate averages over the various energy bands.

EXAMPLE 3–2 Find the $(E, \mathbf{k})$ relationship for a free electron and relate it to the electron mass.



SOLUTION

From Example 3-1, the electron momentum is $\mathrm{p} = m\mathrm{v} = \hbar\mathbf{k}$. Then

$$E = \frac{1}{2}m\mathrm{v}^2 = \frac{1}{2}\frac{\mathrm{p}^2}{m} = \frac{\hbar^2}{2m}\mathbf{k}^2$$

Thus the electron energy is parabolic with wave vector $\mathbf{k}$. The electron mass is inversely related to the curvature (second derivative) of the (E, $\mathbf{k}$) relationship, since

$$\frac{d^2E}{d\mathbf{k}^2} = \frac{\hbar^2}{m}$$

Although electrons in solids are not free, most energy bands are close to parabolic at their minima (for conduction bands) or maxima (for valence bands). We can also approximate effective mass near those band extrema from the curvature of the band.

The effective mass of an electron in a band with a given (E, $\mathbf{k}$) relationship is found in Example 3-2 to be

$$m^* = \frac{\hbar^2}{d^2E/d\mathbf{k}^2} \tag{3-3}$$

Thus the curvature of the band determines the electron effective mass. For example, in Fig. 3-6a it is clear that the electron effective mass in GaAs is much smaller in the direct Γ conduction band (strong curvature) than in the L or X minima (weaker curvature, smaller value in the denominator of the m^* expression).

A particularly interesting feature of Figs. 3-5 and 3-6 is that the curvature of $d^2E/d\mathbf{k}^2$ is positive at the conduction band minima, but is negative at the valence band maxima. Thus, the electrons near the top of the valence band have *negative effective mass*, according to Eq. (3-3). Valence band electrons with negative charge and negative mass move in an electric field in the same direction as holes with positive charge and positive mass. As discussed in Section 3.2.1, we can fully account for charge transport in the valence band by considering hole motion.