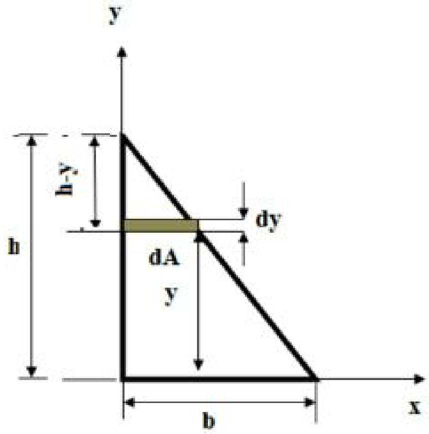
Propriétés Géométriques des Surfaces Planes

Exercice 1

a / Surface $A = \iint dA$

$$dA = xdy$$

$$\frac{x}{h-y} = \frac{b}{h} \qquad x = \frac{b(h-y)}{h}$$



$$dA = xdy = \frac{b(h-y)}{h}dy$$

$$A = \int xdy = \int_0^h \frac{b(h-y)}{h}dy = \int_0^h bdy - \frac{bh}{h}dy$$

$$A = bh - \frac{bh}{2} = bh/2$$

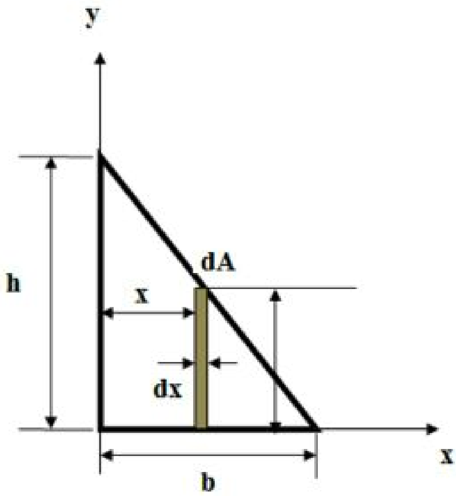
b/ Moments statiques Sx , Sy

$$\mathrm{S_x} = \int \mathrm{ydA} = \int_0^{\mathrm{h}} \mathrm{yxdy} = \int_0^{\mathrm{h}} (\mathrm{h}-\mathrm{y})\frac{\mathrm{b}}{\mathrm{h}}\mathrm{dy}$$

$$\mathrm{S_x} = \int_0^{\mathrm{h}} \left(\mathrm{yb} - \frac{\mathrm{by}^2}{\mathrm{h}}\right)\mathrm{dy} = \left[\frac{\mathrm{by}^2}{2} - \frac{\mathrm{by}^3}{3\mathrm{h}}\right]_0^{\mathrm{h}} = \frac{\mathrm{bh}^2}{6}$$

$$\mathbf{S_x} = \frac{\mathbf{bh^2}}{\mathbf{6}}$$

$$S_y = \int xdA = \int_0^b xydx$$



$$S_y = \int_0^b x\frac{h}{b}(b-x)dx$$

$$\frac{y}{(b-x)} = \frac{h}{b}$$

$$S_y = \frac{h}{b}\int_0^b (bx - x^2)dx$$

$$S_y = \frac{h}{b}\left[\frac{bx^2}{2} - \frac{x^3}{3}\right]_0^b = \frac{h}{b}\left[\frac{b^3}{2} - \frac{b^3}{3}\right] = \frac{hb^2}{6}$$

$$\boldsymbol{S_y = \frac{hb^2}{6}}$$

C/ Coordonnées du centre de gravité

$$x_G = \frac{S_y}{A} = \frac{\frac{hb^2}{6}}{\frac{hb}{2}} = b/3$$

$$y_G = \frac{S_x}{A} = \frac{\frac{bh^2}{6}}{\frac{hb}{2}} = h/3$$

d/ Moments quadratiques

$$I_x = \int y^2 dA = \int_0^h y^2\, xdy = \int_0^h y^2 \frac{b(h-y)}{h} dy = \frac{bh^3}{12}$$

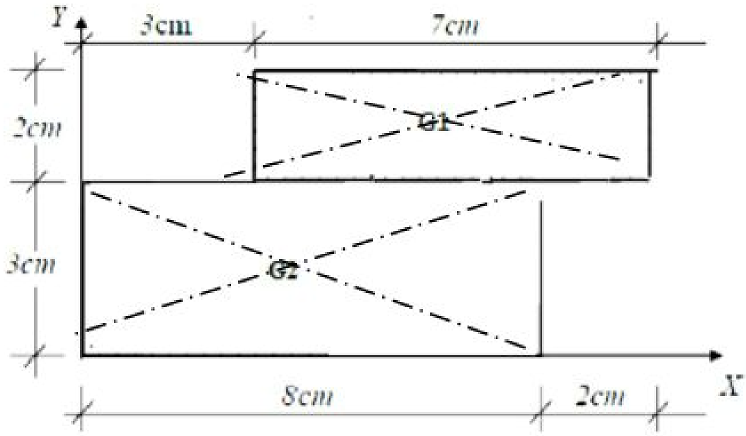
$$I_y = \int x^2 dA = \int_0^b x^2\, xdy = \int_0^b x^2 \frac{h(b-x)}{b} dx = \frac{hb^3}{12}$$

$$I_{xy} = \int xydA = \int_0^h xy\, xdy = \int_0^h \frac{b(h-y)}{h} y \frac{b(h-y)}{h} dy = \frac{b^2}{h^2} \int_0^h (h^2 - 2hy + y^2) y\, dy$$

$$I_{xy} = \frac{b^2 h^2}{12}$$

Exercice 2

$$\boldsymbol{x_G = \frac{S_y}{A_{totale}}}$$



$A_{totale} = A_1 + A_2 = 8.3 + 2.7 = 38\ cm^2$

$S_y = X_{G1}A_1 + X_{G2}A_2$

$x_{G1} = 4cm \qquad x_{G2} = 3 + 3{,}5 = 6{,}5\ cm$

$S_y = 4.(8.3) + 6{,}5.(2.7) = 187\ cm^3$

$$x_G = \frac{187}{38} = 4{,}92cm$$

$$\boldsymbol{y_G = \frac{S_x}{A_{totale}}}$$

$S_x = y_{G1}A_1 + y_{G2}A_2$

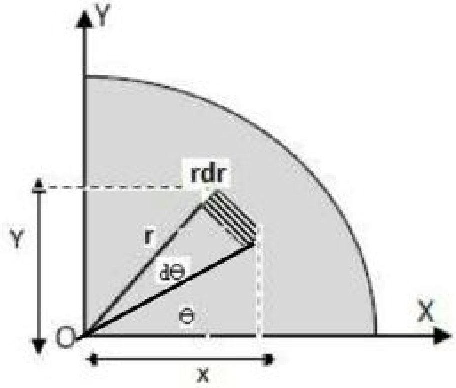
$y_{G1} = 1{,}5cm \qquad y_{G2} = 3 + 1 = 4\ cm$

$S_x = 1{,}5.(8.3) + 4.(2.7) = 92\ cm^3$

$$y_G = \frac{92}{38} = 2{,}42cm$$

Exercice 3

a/surface



$$A = \int dA = \int_0^R \int_0^{\frac{\pi}{2}} r d\theta dr$$

$$A = \int_0^R [\theta]_0^{\frac{\pi}{2}} r dr = \frac{\pi}{2}\int_0^R r dr = \frac{\pi}{2}\left[\frac{r^2}{2}\right]_0^2$$

$$A = \frac{\pi R^2}{4} = \frac{\pi D^2}{16} \quad avec \ R = \frac{D}{2}$$

b/les moments statiques

$$S_x = \int y dA = \int_0^R \int_0^{\frac{\pi}{2}} y r dr d\theta$$

$$y = r\sin\theta$$

$$S_x = \int_0^R \int_0^{\frac{\pi}{2}} (r\sin\theta) r dr d\theta = \int_0^R [-\cos\theta]_0^{\frac{\pi}{2}} r^2 dr$$

$$S_x = -\int_0^R [0-1] r^2 dr = \frac{R^3}{3}$$

$$S_x = \frac{R^3}{3}$$

Par symétrie $S_x = S_y = \frac{R^3}{3}$

$$x_G = y_G = \frac{S_x}{A} = \frac{R^3/3}{\pi R^2/4} = \frac{4R}{3\pi}$$

C/ moment d'inertie

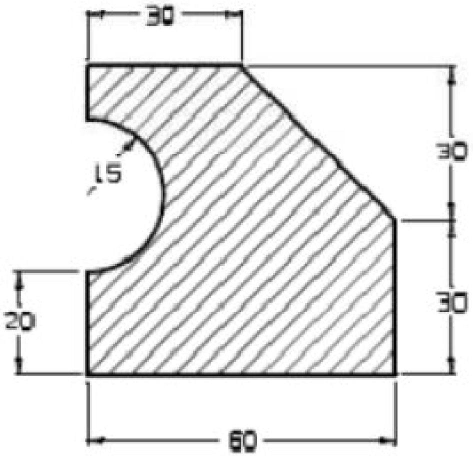
$$I_x = \int y^2 \, dA$$

$$I_x = \int_0^R \int_0^{\pi/2} (r^2\sin^2\theta) \, r dr d\theta = \int_0^R \int_0^{\pi/2} (\sin^2\theta) d\theta \, r^3 dr$$

$I_x = \frac{\pi R^4}{16} = \frac{\pi D^4}{256} = I_y$ par symétrie

$$I_P = I_x + I_y = \frac{\pi D^4}{128}$$

Exercice 5

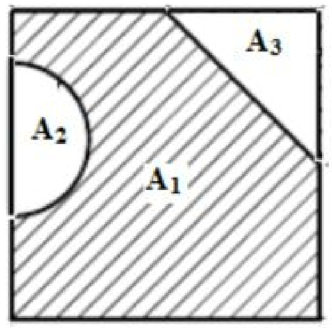


$A = A_1 - A_2 - A_3 = 2796{,}571\ mm^2$

$A_1 = 60.60 = 3600\ mm^2$

$A_2 = \frac{\pi(15)^2}{2} = 353{,}429\ mm^2$

$A_2 = \frac{30.30}{2} = 450\ mm^2$



$S_x = S_{x1} - S_{x2} - S_{x3}$

$S_{x1} = y_{G1}A_1 = 30.(60.60) = 108\,000\ mm^3$

$S_{x2} = y_{G2}A_2 = 35.(A_2) = 12\,370{,}021\ mm^3$

$S_{x3} = y_{G3}A_3 = \left(30 + \frac{2}{3}(30)\right)(A_3) = 22\,500\ mm^3$

$y_G = \frac{S_x}{A_{totale}} = \frac{73\,129{,}98}{2796{,}571} = 26{,}14\ mm$

$S_y = S_{y1} - S_{y2} - S_{y3}$

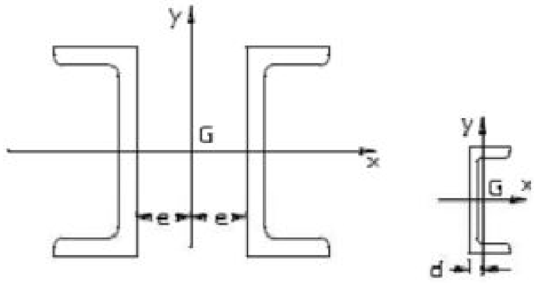
$S_{y1} = S_{x1} = 108\,000\ mm^3$ (symétrie)

$S_{y2} = x_{G2}A_2 = \frac{4R}{3\pi}.(A_2) = 2250\ mm^3$

$S_{y3} = x_{G3}A_3 = \left(30 + \frac{2}{3}(30)\right)(A_3) = 22\,500\ mm^3$

$x_G = \frac{S_y}{A_{totale}} = 29{,}76\ mm$

Exercice 6



A=42.3 cm^2
d=2.23 cm
I_x=3600 cm^4
I_y=248 cm^4.

Nous savons que pour une surface

$$I_x = I_{xG} + y_G^2 A$$

L'axe x est un axe de symétrie pour chaque pièce

$$I_x = I_{xG1} + y_{G1}^2 A_1 + I_{xG2} + y_{G2}^2 A_2$$

$$A_1 = A_2$$

$y_{G1} = y_{G2} = 0$ (Axe de symétrie)

$I_{xG1} = I_{xG2}$ (Même surface + même disposition par rapport à l'axe x)

$$\boldsymbol{I_x = 2.I_{xG1}}$$

$$I_y = I_{yG1} + x_{G1}^2 A_1 + I_{yG2} + x_{G2}^2 A_2$$

$$A_1 = A_2$$

$$x_{G1} = x_{G2} = (e + d)$$

$$I_{yG1} = I_{yG2}$$

$$\boldsymbol{I_y = 2.I_{yG1} + 2(e + d)^2 A_1}$$

$$\boldsymbol{I_x = I_y}$$

$$I_{xG1} = I_{yG1} + (e + d)^2 A_1$$

$$I_{xG1} = I_{yG1} + (e^2 + 2ed + d^2) A_1$$

$$e^2 + 2ed + d^2 + {}^{I_{yG1}}/_{A_1} - {}^{I_{xG1}}/_{A_1} = 0$$

$$e^2 + 2e(2{,}23) + (2{,}23)^2 + {}^{248}/_{42{,}3} - {}^{3600}/_{42{,}3} = 0$$

$$e^2 + 4{,}46e - 74{,}27 = 0$$

$$\Delta = (4{,}46)^2 - 4(1)(-74{,}27) = 316{,}97$$

$$e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{-4{,}46 \mp \sqrt{316{,}97}}{2.(1)}$$

$e_1 = -11{,}13\ cm$ Rejeté

$e_2 = 6{,}66\ cm$ Accepté

$$\boldsymbol{e = 6{,}66\ cm}$$